Joint Workshop on Standards for the Use of Models that Define the Data and Processes of Information Systems April, 22-25, Bellevue, Washington, USA

## System Modelling

PhYsical concepts, ThEORETICAL ELEMENTS, DESCRIPTION LANGUAGE

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## Part 1 <br> EXECUTIVE SUMMARY

Formerly, this contribution dealt essentially with the ISO project $n^{\circ} 1.21 .63$ : this project concerns a « standard for the Conceptual Schema Modelling Facilities» (CSMF standard). It appears further that the items of the proposed methodology, outlined thereafter, may concern also some other ISO activities. Thus, this observation justifies the integral presentation of this study to the ISO meeting in Bellevue.

This study, all the time in improvement, was undertaken several years ago, with the technical or financial assistance of:

- Laboratoire Central des Ponts et Chaussées (Ministère de l'Équipement) ${ }^{1}$,
- Scétauroute ${ }^{2}$,
- Club des Utilisateurs Bull Européens ${ }^{3}$.


## A) Need of CSMF

System designers and users, in any organisation, have skills normally covering fields other than information technology; these fields constitute their Universe of Discourse (UoD). Consequently, if information technology is of unquestionable importance in any enterprise, it must always be combined with other techniques implemented for the same purpose with equal importance; too radical a separation between these other techniques and information technology, more particularly with regard to software engineering, would be unrealistic. The desired organisation calls for the availability of a method and practical facilities for assistance in the use of information technology in their current activities, in particular for the specification of the functions they assign to the various devices, and specially to the computers, within the systems concerning them.

The engineer having overall responsibility for IT equipment in his enterprise can only have a general attitude with regard to the diversity of techniques and domains involved; however he must have methodical and technical aids enabling him to act as an informed interlocutor of contracting firms, which are no less diverse in their respective specialities, and who he will have to call upon. Consequently, we have chosen the hypothesis of an engineer specialising in a particular engineering domain but not a specialist of the many techniques concerning information technology.

Every engineer required to work on a system or on a component of a system, obviously needs any information existing on this item when performing his work. In addition, information concerning the rest of the system may also be useful to him. Thus, before doing any action on the system, he must consult a volume of information whose size increases with the size and with the complexity of the system

Simulation is the technique usually employed up to the present time to aid engineers in certain types of work. This technique, used for many years, long before the advent of data processing, makes use of means which are either created empirically or deduced by analogy or by the transformation of theoretical models of the items to be simulated. Data processing subsequently favoured the use of numerical simulation, since any program intended for calculation on the values of state parameters of any real item constitutes, in fact, a numerical simulation of this item.

And, the purposes of a theoretical model involve aspects other than numerical simulation, even if data processing has found in this area a privileged field of application: other uses of models are possible. These uses constitute the domain of «Computed Aided X » where X represents different possible activities: design, maintenance, technician teaching, fault diagnostic... From this viewpoint, it should be noted that the notion of model has taken on a new connotation. Formerly, a model, expressed in the form of a program, was a technique intended for data processing. It has quickly become data itself, to be processed by techniques developed for aiding engineers in their activities ${ }^{4}$.

These objectives depend on the existence of strict modelling facilities: the CSMFs which provide conceptual definitions, language and software tools (Figure 1).

[^0]

Figure 1: Relation within the CSMF tryptique and its uses

## B) Need of standard for CSMFs

The development of relationships between organisations involves a correlative elaboration of links between their systems. There is the origin of the concept of open system.

At any time, a manager may have to guarantee the safety of his system; nowadays, the idea of a totally closed system, i.e. isolated from its environment, is an abstraction. On the contrary, the daily extension of the collection of functions of this system outward, by linking one or more surrounding existing systems, becomes usual.

The accumulation of these exchanges between systems requires the development of common practices for facilitating them.

## C) Structure of the study

The next part «Physical concepts » determines, in a an natural language, an unitary and reduced system of concepts and method for producing assemblages of concepts; however, all the items of this approach may be modelled with the further theoretical elements. So, and because mathematics have the main advantage of providing strict models, the following part « Theoretical elements» proposes a structured collection of mathematical elements as a fundamental instrument for this general purpose. Let us note that this approach complies with IRDS ${ }^{5}$. The last part «Description language » determines a short formal etymology method for expressing the precedent concepts; for fixing their axiomatic origins or their relative positions in relation to other concepts; for allowing the comparison between them and other different existing definitions, and finally for introducing a formal definition method of description languages.

The two parts « Physical concepts » and « Description language » show a possible way for the CSMF standard study. The part «Theoretical elements » belongs to mathematics; thus it is universal and has not to be standardised; nevertheless it shows how understandable elementary mathematical concepts may support IT standard without strenuous elaboration. The Figure 2 shows the relationships between the different parts of the study and indicates those of these parts which may belong to a standard document (dashed surfaces).


Figure 2: Structure of the study

## Part 2 <br> Physical Concepts (Tin frame)


#### Abstract

Pourquoy est ce que nostre langage commun, si aysé à tout aultre usage, devient obscur et non intelligible en contract et testament; et que celuy qui s'exprime si clairement, quoy qu'il die et escrive, ne trouve en cela aulcune maniere de se declarer qui ne tumbe en doubte et contradiction ?... Montaigne, Les Essais, De l'experience.


$\Delta$ Multiple definitions. Some other similar definitions of the concepts described hereafter are collected in Appendix I. They are stated in different normative documents. If necessary, their existence is pointed out with the symbol heading this note.

It is upon the establishment of some universal simple concepts that lies our only chance of standardisation of system description. The notions developed below, necessarily includes the plurality of techniques used; it is in relation with this broader acceptance of them, that the notion of integration must be considered. They also take in account the plurality of UoD such as: banking, transportation, industrial process control, research, etc.

In that order, the main purpose of this informal chapter is to propose some basic, neutral and general concepts having the theoretical elements proposed in the following part as sound foundations. The principles of their current expression will be examined in the next part. The choice and the definition of basic concepts are justified by common and global needs of system designers dealing with different fields of activities.

## 2 <br> DEFINITIONS OF PRIMARY COMPONENTS OF SYSTEMS

## 2.1 <br> General

## Multiple definitions: object (Page 55)

The concepts are expressed there from an user's viewpoint. Some will at times appear naive to readers. This apparent naivety is however deceptive because the assemblies subsequently constructed using these notions are often complex; a property of information technology resides in the fact that it favours the growth of systems in which the operating logic surreptitiously exceeds human understanding. It is thus important to pay rigorous attention to the definition of basic notions, even the simplest or most evident, thus hopefully avoiding the risk of subsequent ambiguities.

A terminological complication appears. First, the observable components belong to the real world supporting the UoD. Nevertheless, each time we have to study such a component, we must use a representation: the need to describe a component arises just when it comes into consideration for study or for use. A representation may be: a description in a natural language, an image, a model using a sound formalism, etc. Any representation now is also a component and so forth. For that reason the word «component» has a wider acceptation: the same word may be used for speaking of a real entity or for its representation. However, any activity which may concerns us, produces essentially component descriptions, at different stages of completion, and not the real components themselves. Then, in the absence of ambiguity and to avoid abusive language, the
term « component» will be used in place of the corresponding expression « component description ». The same assumption is done for: system, module, process, produce ${ }^{6}$, etc.

## 2.2

System

### 2.2.1 General definitions

Multiple definitions: system (Page 55)
A system is by convention specific to a domain of activity; it is a dependent part of real world.
concept 1: A system is a collection of components called devices (concept 2), individually processing and exchanging other components called produces (concept 3), with each other and with the environment of the system ${ }^{7}$, in order to achieve the execution of a production or of a service of an industrial nature:
concept 2: A device is a component of a system or a part of a system; it represents a set of coherent functionalities (concept 4) ${ }^{8}$.
concept 3: A produce is a component handled by a device as input or output of its functionalities or exchanged between devices.

This distinction between device and produce is arbitrary: we will further show that a device may be also viewed as a produce and reciprocally. One may object that there is no need for two words to express the same concept and, that there is a vain terminological complication. In spite of these remarks, this terminological distinction will be justified further by the distinction of components having different positions in the system.

### 2.2.2 <br> General case: physical system

For different purposes, every physical system must be analysed. This analysis is related to the formal system chosen by the analyst, and enabling the result to be expressed in the form of a description ${ }^{9}$. This descriptive aspect, though closely related to the analytical aspect, will sometimes be examined separately in this chapter.

This chapter deals with the general case of real systems integrating elements of information technology, without being limited to systems made up exclusively of such elements. For the designer of a system or its subsequent operator, knowledge of the system and of the physical laws governing it must prevail over that of the elements of information technology used as tools. The notions previously introduced and all that which will be introduced later are usual in certain fields of activity, such as conversion industries. They are generalisable: an automated shop is thus analysable, but so also is the entire company in general and any software in particular. Examples 1 to 12 beneath illustrate the diversity of the types of physical systems, from a top level to a very low level, falling within the scope of this chapter.

6 In this version, we prefer to use the word «produce» rather than the word «product» when we are speaking of physical system element. This choice eliminates the polysemy due to the use of the physical acceptation and the mathematical acceptation of « product » in the same paper.
7 Observing the outside, every user module is generally affected by the existence of an environment with which it is destined to interact, but which escapes the control of the system manager. The idea of a totally closed system, i.e. isolated from its environment, is an abstraction: the negation of any exchange of a given system with the rest of the world is a working hypothesis which is always possible but dangerous, especially when its safety is specified. The extension of the system outward by introducing one or more surrounding modules results from this requirement. The specification of these modules is most often equivalent to describe passively the constraints they impose, sometimes outside of standards.
8 Preference is given to the neologism "functionality" over the term "function" whose sense, fixed strictly in mathematics and formal linguistics, is too restrictive for general use.
9 The use of several formal systems is current practice; this is the case, for example, of an algorithm whose expression in a given software exists in several formal systems: high-level language, intermediate machine languages, binary language.

| Example | 1: A network of motorways, with its infrastructure and its appurtenances, its users and its information system. |
| :---: | :---: |
| Example | 2: A multi-purpose vehicle for the measurement of pavement properties. |
| Example | 3: A bridge deck. |
| Example | 4: A bridge deck raising site. |
| Example | 5: An information system. |
| Example | 6: A communications network. |
| Example | 7: A microcomputer and its software. |
| Example | 8: An office automation system. |
| Example | 9: An accounting software. |
| Example | 10: A data acquisition and interpretation system. |
| Example | 11: An " intelligent " actuator (hardware and system with built-in microprocessor for control and communication). |

Example 11: An " intelligent" actuator (hardware and system with built-in microprocessor for control and communication).
Example 12: A sensor.
The theoretical elements defined in the next part, are intended to support the modelling of such entities.

The listing of elements distinguished by analysis constitutes the morphology (concepts 17, 19) of the system; the expression of the rules of individual or mutual behaviour of the elements constitutes the physiology (concepts 18,25) or different aspects of the behaviour of the system (concepts 18, 26). A physical system is never isolated: there is at least one process receiving at least one produce from the outside; similarly, there is at least one process sending at least one produce to the outside. This outside, about which there is often scant knowledge, always different from what is known about the physical system, is the result of a generally distinct approach; it is called environment of the system ${ }^{10}$.

The notion of subsystem, also called module, is introduced in this set of basic concepts in order to facilitate the hierarchical classification of components belonging to the same system or to its environment.

Example 13: The part of a system providing communication between all the elements of this system may be set apart as a module.

### 2.2.3 <br> Special case: information technology system

Some modules may be made up exclusively of elements pertaining to information technology. Any module of this type may be isolated conventionally from the rest of the system; it is then called an information technology system, and the physical system within which it is integrated becomes its environment.

In spite of the importance currently attached to these particular elements, it appears clearly that they constitute neither its morphological totality, nor its physiological finality. It is valuable to the architect engineer, at any time in the life cycle of the physical system, to have methods and means of action taking into account the presence of information technology, but which are not restricted thereto. Two Examples show the relative nature of the notion of information technology system. They show the plurality of relations linking these systems and their respective environments and the resulting diversity of standardisation requirements.

Example 14: An information system (Example 5, Page 13) is a possible module in a motorway network (Example 1, Page 13). It is generally made up of elements of information technology and, in this case, considered to be an information technology system.
Example 15: In a motorway network (Example 1, Page 13) for which many data acquisition operations are to be carried out using appropriate instrumentation, every acquisition and interpretation chain (Example 10 Page 13) is assimilable to an element integrated in the operating equipment (toll, signing, control, etc.) and specific to this equipment. On the other hand, in a bridge deck raising operation (Example 4 Page 13), the chain that acquires and restores in real time to the operator all the information relative to the behaviour of the structure is a sophisticated module considered by the construction project head to be an information technology system.
An information system embedded in any physical system is certainly nowadays one of the masterpieces of this physical system. This kind of module is directly issued from the use of some CSMFs ${ }^{11}$.

[^1]
## $2.3 \quad$ Functionality

### 2.3.1

## Definition

A functionality F is the most elementary form of a device. The use of this neologism is preferred to that of function, because the properties assigned to it are more general than those of the notion of function ${ }^{12}$.
concept 4: The classical diagram of Figure 3 represents graphically three constituent


Figure 3: Functionality
concept 5: its body (M) which is a device,
concept 6: the specification of produces (I) placed at its inputs,
concept 7: the specification of produces ( O ) placed at its outputs.
The functionality moreover supports general conditions or constraints imposed upon it: these are the known physical laws applicable to this functionality:
concept 8: those concerning I are pre-conditions;
concept 9: those concerning $O$ are post-conditions;
concept 10: those concerning both I and O and mutually controlling them are invariants ${ }^{14}$.
The notion of functionality is recursive: any functionality defined after an analysis stage can, in turn, be analysed in new functionalities. At the start, before any analysis work, the entire physical system constitutes itself a functionality. From the simple viewpoint of the approach we are describing, the notions of system and functionality are thus identical; we shall however distinguish them conventionally, leaving the notion of system at the start of the analysis and placing the notion of functionality at the different stages of the analysis.

### 2.3.2

Relations between functionalities: exchange of produces, interoperability

Multiple definitions: interoperability (Page 54)
The interactions between devices of the real physical system are possible only if exchanges of produces exist between these devices. The relations between functionalities describe these interactions. Yet, these interactions and consequently the relations that express them depend on various physical laws or conventions applicable to each


Figure 4: Exchange of produces examined part. The only exception is the

12 In fact, this neologism is totally unnecessary: the algebraic notion of functor accounts for the properties attributed to this diagram. This term could thus be used.
13 This well-known diagram is currently used. Mathematical models are less used. Let us however note that the algebraic notions of category and functor are used for modelling E, S and M. The advantage of this approach is that it accounts in a global and standard manner for the morphological and physiological properties of these terms, whatever their respective physical natures.
14 These constraints often remain implicit - or ignored - in system specifications or descriptions. The reason is that many items of equipment are designed and constructed knowing these laws; they consequently protect the uninformed user from their possibly harmful effects with the reserve that this user indeed complies with the validity limits indicated by the supplier. These validity limits are in most cases broad, leading to a false impression of safety, and then negligence, and finally involuntarily abusive usages involving immediate or latent errors. There are many instances in instrumentation, and simply reasoning by induction enables us, for example, to draw up a sizeable inventory of deleterious effects.
relation, omnipresent in all systems, expressing the existence of an exchange to the exclusion of all other properties, for any couple made up of an output O of a given functionality F and an input I' of any functionality $\mathrm{F}^{15}$.

The specifications of O and $\mathrm{I}^{\prime}$ are generally different but must obviously be consistent: the template of I' must be compatible with that of O. Two questions must be answered:

- define consistency criteria for the specifications between O and I', which criteria do not exist in the absolute, but depend most often on the couple ( $\mathrm{F}, \mathrm{F}^{\prime}$ ),
- provide the practical means of verifying that this consistency exists in the specifications for any exchange, the verification quickly becoming complicated with the number of exchanges ${ }^{16}$. That is one of the major problems of system validation.
concept 11: The consistency of O in F with $\mathrm{I}^{\prime}$ in $\mathrm{F}^{\prime}$ determines the quality of interoperability ${ }^{17}$ of the couple ( $\mathrm{F}, \mathrm{F}^{\prime}$ ).

Let us note in this respect that interoperability, thus defined, is a quality attached to a couple of functionalities and not intrinsic to a single functionality. Yet, there is a lack of definition: this definition of interoperability lies upon the precedent definition of consistency which is not clearly stated above. Here is an example of dubious informal definition which needs the assistance of mathematics for clarifying it.

| Example 16: | F is a sensor (Example 12, Page 13). I is a physical magnitude: the frequency at which vehicles cross a given point of a pavement; M a device such as an induction loop embedded in a pavement and its associated circuits; M delivers a voltage O of extremely low amplitude whose derivative changes sign with a frequency proportional to the value of the magnitude I measured. In other words, $O$ is a train of "unformatted" pulses. It should be noted here that $O$ must be regarded as a DC voltage whose evolution depends only on variations in I and can therefore not obey any standard, by definition and construction. $\mathrm{F}^{\prime}$ is a measurement chain including, among other functionalities, a pulse counter reacting only on the edges of variations in I' and imposing by construction of $\mathrm{M}^{\prime}$ a minimum restoring time between two successive edges. The two templates present are thus totally different, but may be consistent: the rising or falling edges of O must be exhaustively identifiable by $\mathrm{M}^{18}$. |
| :---: | :---: |
| Example 17: | F is the counting device described above (Example 16). O is a structure of data including dates, times and values of counts. $\mathrm{F}^{\prime}$ is an information system (Example 5, Page 13). I' is a structure of digital data made up of fields allowing the recording of a nature of variable, its value, its acquisition attributes. $\mathrm{M}^{\prime}$ is a set of functionalities allowing the acquisition, centralisation, management and finally selection by interrogation of such data structures. O' is a structure identical to I'. Consistency requires, in this case, that the structure O be included in I'. |

2.3.3

Relations between functionalities: translation of a functionality into another, portability
$\boxtimes$ Multiple definitions: portability (Page 55)
The definition of
F: M(I,O)
consists in defining the properties required for each of these elements as well as the constraints. The body M, its inputs I, its outputs O, then have an expression in a formal system S (Figure 5). This expression may be translated in another formal system $S^{\prime}$ :

F': M'(I', O')
$F^{\prime}$ is a new functionality whose properties comply with the conditions previously specified:
concept 12: The specifications of the functionality F to be transported determine a template; the portability of $F$ implies that the features of every implementation must fall within this template.

15 The Graph theory can be used here; it is then important to clearly specify the interpretation adopted for the apexes and the arcs. In the approach of this paragraph, the apexes are inputs or outputs of functionalities. The arcs represent recent exchanges between functionalities but also the functionalities themselves, which establish links between their own inputs and outputs.
16 This consistency check between functionalities (in the specifications) or between components making up these functionalities (in the solutions) calls for formal verification methods. We shall return to this point in the last chapter.
17 Neologism (F: interoperabilité).
18 This Example may be considered as a "butterfly" effect in automatic control. It is a point of detail obviously ignored by owners. It entails a latent risk of inadvertently false measurements having, for example, the real-time result of failure or breakage of equipment and always resulting in financial losses incommensurate with the relatively modest cost of the sensor.

Formally, a good solution consists in proposing such transformations $T, U$, that $I^{\prime}=T(I), M^{\prime}=U(M)$ and such a transformation V that, with its inverse ${ }^{19}, \mathrm{O}=\mathrm{V}^{-1}\left(\mathrm{O}^{\prime}\right)$. As a functionality is a description of a real element of a system and not the element itself, the operation described above can be assimilated with the passage from a first assembly in a formal system to a second assembly, image of the preceding, in another formal system.

This diagram gives an image of portability ${ }^{20}$. Let us assume the existence of a first functionality F which must be implemented in different formal systems $S^{\prime}$, $S$ '', etc. If $F$ ' in $S$ ', F', in S', etc. exist, then F is transportable from $S$ ' to $S$ '', etc.

Example 18: F is a program expressed in a high level language $S$, and $F$ ' is the same program translated into the language $S^{\prime}$ of the computer.


Figure 5: Implementation

### 2.3.4 Interface

$\boxtimes$ Multiple definitions: interface (Page 54)
The preceding discussions show the importance of the notion of consistency in the definition of system specifications as well as in the synthesis of a solution. The questions of consistency remain when reference is made to standards for specifications, and when elements of standardised solutions are used during construction. The notion of interface is developed in Standardisation to answer these questions.

The interface is, by convention, a set of specifications determining the characteristics that must be met by functionalities in order to proceed with exchanges. Two types of interfaces thus correspond to the two types of relations determined in the first two chapters.
a) Interface of the first type
 tionalities F and F'. In addition to the definition of the structured set of informations to be linked ( O and I'), these conventions must also, if required, include time-dependent specifications as well as qualitative specifications (e.g.: safety) concerning this information.
Let us note that the lack of consistency between O and I' occurs frequently in heterogeneous system. This implies the insertion halfway to M and $\mathrm{M}^{\prime}$ of a mechanism which is well a component which ensures the compliance of O with $\mathrm{I}^{\prime}$.

[^2]
## b) Interface of second type

concept 14: The interface of the second type (Figure 7) is a set of conventions governing the transformations from an expression $(I, M, O)$ pertaining to a formalism S into a translation (I', M', O') pertaining to an other formalism $\mathrm{S}^{\prime}$. The conventions involve produces I, I' and O, O' to be processed by the functionalities with, in addition the functionalities M , $\mathrm{M}^{\prime}$ themselves handling these informations.
Let us note that in this matter, an component serviceable for performing these transformations, and also if necessary their reverses, must exist.


## $2.4 \quad$ Device and produce

A device is a structured collection of functionalities. It has an interest in a system if and only if it is able to interact with other devices. This condition means that the component has, at least, one facial functionality.
concept 15: A functionality F can be seen from another functionality $\mathrm{F}^{\prime}$ if it may send at least one produce towards this functionality or if it may receive at least one produce from it. F is a facial functionality if F' may belong to another component.
concept 16: The collection of facial functionalities of a device constitutes its face or its visible part.
This definition is simple. It assimilates an device to a black box having some points of exchange with its environment. A produce is a component exchanged between devices or performed by a device. A produce may be perceived as a functionality in its simplest form considering the physical absence of input and output specification. A device may be considered as a produce and exchanged between devices or performed by an other device. Inputs and outputs of functionality are produces.

Example 19: A program or data exchanged between two computers.
Example 20: A program processed by a translator.

## $3.1 \quad$ General

In treating of any part of a system, we must deal with it in two ways. We must deal with its observable structure made of heterogeneous components and its possible variations in time; we must also deal with its present and future possible uses. That is to say we have to study it morphologically and also physiologicallly.

Two ways for system designing are examined: analysis (top-down) and synthesis (bottom-up).

### 3.2 Analysis

### 3.2.1 <br> General definitions

## Multiple definitions: behaviour (Page 54)

The analysis of a system or of a device makes it possible to distinguish iteratively, gradually, the devices and produces composing it. Let us summarise the basic elements of the approach. A system is a coherent whole of parts assembled to achieve a common goal. The first step in the analyse of a system is the separation of components and the recognition of relationships between them. After, each component may in turn be regarded as a system. The model and its expression must express this general structural aspect. Correspondingly we agree to build our models in two parts: morphology and physiology.
concept 17: the definition of a components collection which compose it; these components are defined globally without analysing their respective details, outside of the inventory of the produces they are liable to exchange; we name this collection: morphology (concept 19) ;.
concept 18: the definition of rules collection which determines the interactions between these components; these rules constitute the behaviour of the analysed system; we name also this collection: physiology (concept 25);

### 3.2.2 Morphology

concept 19 The primary morphology of a system includes the following elements, which are all components of the system:
concept 20: produces;
concept 21: devices used for handling the flows of produces or converting these produces.
Each produce may in turn be regarded as a system having iteratively at its level a primary morphology and so on...


Figure 8: Component
concept 22: the exhaustive morphology results from this primary morphology, and in addition from the primary morphology of the different components belonging to it and so on... . In order to simplify the text, a primary morphology will be called shortly morphology.

Example 21: Flows of energy, materials, information, programs, and so on.
Example 22: Construction site equipment, industrial computer interfaces, programmes, and so forth.
Let us note that certain elements may belong to both categories mentioned above. Finally,
concept 23: some elements of a morphology are known from outside the device and constitute its visible part,
concept 24: while others constitute its internal part.
Example 23: A sensor, an actuator may be viewed as elements of the visible part of a morphology.

### 3.2.3

 PhysiologyMultiple definitions: action (Page 54)
concept 25: A primary behaviour (or primary physiology) is a collection of actions
concept 26: the exhaustive behaviour (or exhaustive physiology) results from this primary physiology, and in addition from the primary physiology of the different components belonging to it and so on...

Each element of a physiology, called an action specification, must specify the conditions for activating each device and must express, among other conditions, the time-related constraints to which it is subjected.

### 3.2.4 <br> State of a component <br> $\triangle$ Multiple definitions: state (Page 55)



Figure 9: State, change of state
The morphology and the physiology of an component constitute both a general description of all the possible manifestations of this component.
concept 27: A description of a particular manifestation of the component is called an occurrence.
concept 28: An occurrence, expressed within a translation with respect to time, is called: limit sate of the component.
concept 29: A sample is a part, during any time interval, of any limit state of the component.
The set of all possible limit states is associated with the component. This set is structurable (Figure 9). In particular, two main classes of parts may be distinguished:

- the part made of limit states usually pertinent for the UoD,
- the complementary part.
concept 30: Every subset of the class of pertinent limit states is called a state of the component. Any set or subsets may be represented by one of its elements.
concept 31: Any subset belonging to the complementary part is an exception.


### 3.2.5 Recursivity of morphology and physiology

The general structure above can be used for the analysis or the description of a system or of a device. In each case, the final result appears as a hierarchical description of the system in which each level of the hierarchy provides a detailed description of the components introduced into the morphology of the preceding level.

Each level of the hierarchy consequently introduces descriptions containing details which are added to those expressed in the preceding levels. There is a limit to this procedure: the specificity of the elements introduced at each level is asserted as the analysis progresses in detail; after a certain level of analyse, three cases appear as possible:

- the description remains possible and pertinent in the proposed global formalism,
- the description remains possible in the proposed formalism, becomes inadequate or insufficient owing to the specificity of the new properties introduced,
- the description is impossible, owing to insufficient knowledge about the element, or is considered to be unnecessary.
In the analysis of a heterogeneous system, one should always be in a position to stop its description in the global formalism and to specify if necessary the existence of a description in another formalism. This is a difficult formal problem for the definition of consistent languages.


## $3.3 \quad$ Synthesis

The synthesis is a bottom-up mechanism for designing a system. The synthesis of an component consists:

- in creating a complete morphology (visible and internal parts) by means of previously defined components and if necessary of new components,
- in associating the definition of physiology with these morphologies.

It is then necessary to examine how the combination of these components can function. Several cases may be considered, and we shall mention three essential ones:
(i) The definition of components is sufficient for the complete definition of the association (Figure 10, 1st case).
(ii) The definitions of components are insufficient to ensure a complete definition of their association; the definitions are completed by the definition of a supplementary component which, associated with the initial components, brings us back to the preceding case (Figure 10, 2nd case). In this case, the exhaustive behaviour of the whole results from the physiologies of the embedded components.
(iii) The complement to the definitions is provided by a new specific physiology (Figure 10, 3d case); let us note in this case that the whole behaviour of the new component results not only of this new physiology but also from the physiologies of the embedded components.
(iv) The definition of the new component results from a combination of the former cases.


Figure 10: Synthesis of component

Example 24: A collection of asynchronous parallel tasks (1st case).
Example 25: A collection of synchronised parallel tasks; $\Omega$ is a clock (2nd case).
Example 26: A collection of scheduled tasks. The physiology rules the schedule (3d case).

## $3.4 \quad$ Component typology

M Multiple definitions: class (Page 54), type (Page 55)
concept 32: Component typology consists to allow the expression of collectivising relationships between components to be defined later.
Two ways of collectivisation ruling are possible:

- Enumeration and description of each state available for all the components belonging to the set.
- Expression of the laws defining the states and allowing the construction or the recognition of any component having states in compliance with these rules.
concept 33: The collectivisation rules determines a type.
We may associate with any type, the set of all possible components complying with the chosen type.
concept 34: Any existent component belonging to a system is an instantiation of the type with which this component complies. This component is also an element of the set associated with its type.
concept 35: A formal component is a component defined away in state and time and place, that means independently of any actualisation
concept 36: An actual component is the specification of a discernible possible occurrence: the use of this component implies the discrimination of a recognisable state, and rigorous locations in time and in space.
concept 37: Sometimes a particular component may be chosen in order to define and further to represent the type: it is a prototype. In this case the typology must associate to this component, the collectivisation rules necessary for defining the type.
concept 38: The morphology of a prototype is a protomorphology.
concept 39: The physiology of a prototype is a protophysiology.


Figure 11: Use of typology and of genealogy

## Component genealogy

Multiple definitions: inheritance (Page 54)
concept 40: Component genealogy to define a type and to use it as a root or as a parent for further definitions using inheritance rules.
Example 27: An analogue voltage signal is an Example of type of component. The chosen prototype may be the particular signal used for the tuning of apparatus. The sampling is an Example of generic rule. In this case, the descendant type is the set of sampled voltage signals.
Example 28: Let us note that from this approach, the classical subtyping in programming language is a particular and limit aspect of generation; in this case, the generic rules are used only in order to restrict the previous set of states.
concept 41: A parentis an initial or intermediate item of a genealogy used as a basis for applying the inheritance rules
concept 42: The morphology of a parent is a parent-morphology
concept 43: The physiology of a parent is a parent-physiology
concept 44: A descendant is a final or intermediate item of a genealogy which results of an application of the inheritance rules
concept 45: The morphology of a descendant is a descendant-morphology
concept 46: The physiology of a descendant is a descendant-physiology
A particular component may be chosen in order to represent the type. In this case, this component is a generic component. These rules of generation may concern either the morphology, either the physiology. The result of application of generic rules is ordinarily a type. However, the set associated with the type may be reduced to a singleton and used implicitly as a single component.

## Part 3 <br> THEORETICAL ELEMENTS (GOLD FRAME)


#### Abstract

Et ce moyen est le projet que j'ai d'une langue ou écrite nouvelle ... ; et non seulement on trouverait là-dedans des voies infaillibles pour arriver à la solution des problèmes qui peuvent se résoudre par la seule force du raisonnement, mais lors même qu'il s'agit d'une question de fait, et qu'il reste encore des expériences à faire qui ne sont pas toujours dans le pouvoir des hommes, ce calcul serait suffisant pour nous conduire, en attendant, sur les connaissances déjà données. Leibnitz, lettre à Jean Frédéric, 1679.


## Prolegomena

Physical systems are generally described in terms of operations relating physical items. The logical complexity of actual systems calls for simple descriptions in terms of symbolic expressions modelling these items and their relationships. Mathematics and particularly algebra, deals with this purpose.

This part concerns the use of algebraic notions, and particularly those of category theory, in the modelling of systems and, more particularly, in the modelling of those belonging to the field of application of ISO standardisation work.
a) Modelling and vocabulary

The modelling method proposed concerns the representation of systems made up of entities belonging to the physical world by means of systems made up of abstract entities belonging to mathematics. The changeover from one to the other is based upon the use of hypotheses adopting, for the physical units, properties assigned to the entities of mathematics.

The words designating the entities of the physical world ${ }^{21}$ are of current usage; they are however chosen, to the extent possible, preferably from a vocabulary not customary to standardisation; this is seemingly complicated, but the advantages will appear subsequently when we shall apply the results of this work to standardisation work in progress.

Words designating abstract entities are those set by usage in mathematics.


Figure 12: Purpose of the study

We use them in this note without any restriction on their mathematical meaning. This option unavoidably leads to risks of polysemy, as the modelling concerning the domains use these words with different meanings. A fearsome example is that of the "object" word wich has different acceptations in different domains: a component of a category in algebra, a tangible thing of the real world, a concept for system designing in data processing, etc. These situations are dealt with respectively as they appear.

Words designating the modelling elements are introduced by the above-mentioned hypotheses or by definitions.

Axioms and theorems appearing in appendices come from texts cited in the references.

## b) Modelling and graphic representation

Figures accompany the text. They allow fast and intuitive interpretation of the abstract mathematical notions developed. Although consistent with the text, they are sometimes fundamentally restrictive. Recommended to the reader who wishes to conserve a general idea of the proposed approach, they are not recommended for those wishing to use and possibly go beyond the reasoning proposed. For the informed reader, there are, for these figures, approximately the same restrictions as those existing in the particular case of multilinear algebra and its representations in affine Euclidean spaces.

## c) Purpose and limits of theoretical elements

This part proposes modelling elements and, consecutively, a method for model construction. Unlike the usage which is unfortunately widespread, this is not one model, which would thus be universal for the investigated domain: there are in fact potentially many more possible models than entities.

The diversity of the entities of the physical world counter the working out of modelling elements making it possible to depict all their physical properties. The properties of these elements must be sufficiently general to belong to all the entities subject to modelling. Under these conditions, these modelling elements allow the building of models in which is depicted only the existence of these entities and their relations: these are global models.

Finally, the finer modelling of each entity requires the introduction of additional hypotheses specific to it. Generally, these hypotheses and the way they are used to build detailed models belong to often interrelated existing theories: information theory, signal theory, mechanical theory, etc. It is important for the elements introduced in this study to be consistent with these theories. The choice of these elements is delicate: if too general, they would limit our possibilities to the mere construction of models low in properties, i.e. naive and of no interest; if too detailed, they would also limit our possibilities to certain technical areas too restrictive to use. Between these two extremes, the reasonable choice is difficult to establish. This note is thus only a prologue to future developments.

This part is independent of any computer-readable language used for system descriptions. Yet, it supports the formal semantics of such language.

## d) Brief introduction to main notions

The word "system" is frequently used in this note. Its prior definition would be fitting here. However, as the entire note is devoted to the mathematical modelling of systems, this prerequisite will be limited, in the following paragraph, to a statement of simple facts of current observation.

Physical systems are made up of entities working interactively in order to achieve a common technical or scientific goal. Each of these entities is a device. The device perform processing on other entities; each of these other entities is a produce. Devices and produces are the components of the system. Exchanges of produces between devices are indispensable for the existence of interactions between devices. We shall assume for the moment that an entity can be seen as a device or as a produce.

## e) Brief introduction to method

The modelling of a physical system and, step by step, of each entity composing it generally includes two complementary parts:

- The inventory of the entities making up the system and their relations. This is the morphology of the system.
- The inventory of rules governing interactions between entities. This is the physiology of the system.
Analysis determines these top-down inventories; it is recursive: each morphology designates entities; each of these entities is also a system amenable to study and so on. The stopping of the study on an entity is imposed either by a decision on the part of the analyst or by a lack of knowledge.

The reverse construction consisting in assembling entities and governing their interactions to form from bottom to up a new entity is a synthesis. Generally, the modelling of a system is a succession of analyses and syntheses.

An entity whose morphology and physiology are determined is a procedure. A procedure is a model since it is made up of two descriptions: a morphology and a physiology. It is a global and predictive $\underline{\underline{\text { model }} \text { of a physical system, of a device or a produce }{ }^{22} \text { concerning all their possible occurrences. }}$

22 The systems, machines and produces are entities of the physical world, the process is the general model thereof.

The concrete or experimental study of a physical system is a succession of observations of this system or of certain entities of which it is composed during operation. We shall call process the description of an operating case of a procedure. It is a special model of a physical system or device or produce ${ }^{23}$ concerning only one of its possible occurrences.

## f) Summary of approach

Chapter 2: Primitive elements; introduces the theoretical elements necessary in mathematics relative to a process and then a procedure.
Chapter 3: Recursive elements; constructs on the preceding elements the mathematical models of the simple components; it then gives the basic rules for construction by successive analyses or syntheses the mathematical models of complex components.

### 2.1 Method and constructive principles

These elements have to do with whatever is immediately perceptible in a system: observable facts or those that intuition or accumulated experience suggest as observable.

The process describes a real or imagined phenomenon but only in time and space: it involves the functioning of the system or of an entity composing this system; the collection of all possible processes constitutes the fundamental space associated with the entity concerned.

General laws exist for this collection of processes; translated mathematically, they allow the structuring of the fundamental space and its transformation into a new space called a substrate. This type of space is the basis for all the modelling elements developed thereafter.


Figure 13: Production of primitive elements

## $2.2 \quad$ Fundamental space

### 2.2.1 General

As we are concerned with observable phenomena, intuition immediately suggests that any process should be expressed by a function allowing space as a variable. Modelling experience however warns us about the known drawbacks of this representation method: there are techniques, and in particular signal processing, for which the functional representation does not readily account for certain physical properties such as, for example, discontinuities ${ }^{24}$.

The proposed approach thus avoids having to set a priori a precise and definitive mathematical form for the process model. We study the fundamental space without considering any particular form for this

[^3]process. The properties of the fundamental space result directly or by deduction from hypotheses allowed for the represented entities. These properties then determine structures for the fundamental space.

### 2.2.2

Properties
Let us consider a physical system S (or a physical entity) with which we associate the following theoretical elements:

## a) Intrinsic properties

hyp. 1: There is a $G=\{g\}$; each $g$ represents a process of $S ; G^{25}$ is the fundamental space.
hyp. 2: There is a particular element $g_{0} \in \mathrm{G}$ representing the absence of process.

## - Comments

The data item for the assembly $G$ is silent as regards the mathematical nature of its elements; there is thus no collectivising property of the processes $g$ allowing the construction of $G$, except for the one of existing. The existence of G is thus allowed intuitively;

The word "functioning" is used in this text in preference to "behaviour" in order to avoid possible confusion, the latter term moreover having many definitions and already being widely used. To be exhaustive, we must thus include, paradoxically, in the meaning of "functioning" the case of total or partial inactivity of $S$ represented by $g_{0}$.

There are few intrinsic properties. This situation is normal in the absence of knowledge regarding the processes: knowledge regarding processes ordinarily results from their observation and hence from the existence of physical entities allowing these observations. The following hypotheses concern these new entities.

## b) Extrinsic properties

The following modelling elements concern physical entities indispensable for observation:

- observation spaces,
- time which is a part of any observation space,
- binary signals ${ }^{26}$ used for observation control.
- Observation spaces
hyp. 3: There is a countable collection of $n$ sets $A_{i}: A_{0}, A_{1}, \ldots, A_{n}$ where $A_{i}=\{$ ai $\} ; A_{i}$ is a space of values ${ }^{27}$ and ai ${ }^{28}$ is a value;
hyp. 4: Any observation set $\mathrm{A}_{\mathrm{i}}$ must have a "minimal" structure; we shall assume that this minimal structure is a relationship of a total order: $\leq$; Consequently, there is an element ai $i_{0}$ such that ai ${ }_{0}$ $\leq$ ai for any ai. ai ${ }_{0}$ is minimum.
def. 1: The scalar product $\mathrm{A}=\{\mathrm{a}\}=\Pi \mathrm{A}_{\mathrm{i}}(\mathrm{i}>0)$ where $\mathrm{a}=(\mathrm{a} 1, \ldots$, ai, $\ldots$, an $)$ is a reference.
- Time
hyp. 5: $\mathrm{A}_{0}=\mathrm{T}$ represents real time; T is isomorphic to the set of real numbers.
hyp. 6: A closed set $\left[t_{1}, t_{2}\right]$ is associated with each $g$ : this is the domain of $g$.

25 The notion of behaviour will be defined later.
26 Still in anticipation of what follows, let us distinguish:

- bivalents signals having any two states,
- bivalent logical signals having two states: "true ", " false ",
- binary signals having two conventional states: 0,1.

27 The index i marks the rank of each observation space $\mathrm{A}_{\mathrm{i}}$ in the collection of observation spaces; used in the notation ai, it indicates that ai belongs to the space $\mathrm{A}_{\mathrm{i}}$ and does not mark any position in this set; the index notation is avoided for this reason.
28 Note that, in the notation "ai", the letter i indicates that the element belongs to the set $\mathrm{A}_{\mathrm{i}}$ and not the place of this element in this set.

- Binary signals
hyp. 7: There is a $\Delta=\left\{\delta\left[\tau_{1}, \tau_{2}\right]\right\}$; any $\delta\left[\tau_{1}, \tau_{2}\right]$ is a model of the state of the rectangular signal associated with the closed set $\left[\tau_{1}, \tau_{2}\right] \subset \mathrm{T}$ : this is the Heaviside step, ascending at instant $\tau_{1}$, combined with the reverse step, descending at instant $\tau_{2}$. T is the domain of $\delta\left[\tau_{1}, \tau_{2}\right]$.
hyp. 8: There is a $\Delta \varepsilon=\{\delta[\tau]\}$; any $\delta[\tau]$ is a model for the high state of the pulse signal associated with the closed set $[\tau \pm \varepsilon \square / \varepsilon \rightarrow 0] \subset \mathrm{T}$; this is Dirac's pulse centred on $\tau$. T is the domain of $\delta[\tau]$.

Anticipating the rest of the elements, hypotheses 7 and 8 above have to do with special processes: binary elementary signals. These are signals having only two possible states, designated conventionally as high and low. Each occurrence of one of these signals is thus the appearance of one or the other of these states. Only the occurrences of high states are used for the moment.

- Comments

Hypotheses 7 and 8 refer to distribution theory. Under these conditions, any rectangular signal $\delta\left[\tau_{1}, \tau_{2}\right]$ can be derived for any $\tau$ of its domain T. In particular, the derivative for the lower bound $\tau_{1}$ is the occurrence of the pulse $\delta\left[\tau_{1}\right]$ at instant $\tau_{1}$ :

$$
\frac{\partial}{\partial \tau} \delta\left[\tau_{1}, \tau_{2}\right]=\delta\left[\tau_{1}\right], \text { for } \tau=\tau_{1}
$$

For the upper bound, the derivative is $-\delta\left[\tau_{2}\right]$; at any other instant, the derivative is 0 . It will be reminded that $\delta\left[\tau_{1}, \tau_{2}\right]$ and $\delta[\tau]$ are not functions and are nevertheless indefinitely differentiable ${ }^{29}$.

### 2.2.3 Algebraic structures

The following hypotheses have to do with the physical properties of the observed system; these hypotheses are represented by algebraic structures.
hyp. 9: There is an application $\bullet \square: \square \Delta_{\varepsilon} \times G \rightarrow G$. This application is the sampling at the instant $\tau$. The image of any g is a sample.
hyp.10: There is an application $\otimes \square: \square \Delta_{\varepsilon} \times G \rightarrow$ A. This application is the quantification at the instant $\tau$. The result is the quantified value of the sample.
def. 2: For any $g$, the process $g^{\prime}=\delta[\tau] \bullet g$ and $g^{\prime} \in G$ is the instantaneous sample of the process $g$ at the instant $\tau$

For any instant $\tau$ not belonging to the domain of $g$, the sample of $g$ at that instant is the sample of $g_{0}$ :
hyp.11: for any $\tau \square \notin\left[\mathrm{t}_{1}, \mathrm{t}_{2}\right],(\delta[\tau] \bullet g)=\left(\delta[\tau] \bullet \mathrm{g}_{0}\right)$
def. 3: for any $\mathrm{g}, \delta[\tau] \otimes \mathrm{g} \in \mathrm{A}$ is the instantaneous value of g at the instant $\tau$.
For any instant $\tau$, the instantaneous value of a process is equal to the quantified value of its sample at the same instant:
hyp.12: $\delta[\tau] \otimes \mathrm{g}=\delta[\tau] \otimes(\delta[\tau] \bullet \mathrm{g})$
Example 29: g being any process, $\delta[\tau] \bullet g$ is what a fast shutter enables us to observe; $\delta[\tau] \otimes \mathrm{g}$ is what is recorded on a film.

- Comments

If $g$ is a function of the time taking on its values in A , signal theory is applicable directly.
hyp.13: The observation is an application $\bullet \square: \square \Delta_{\varepsilon} \times G \rightarrow G$; for any $\delta\left[\tau_{1}, \tau_{2}\right] \in \Delta$ and all $g \in G$ of domain $\left[t_{1}, t_{2}\right]$, there is $g ’ \in G$ such that:

29 E. Roubine. Intrroduction à la théorie de la communication (Introduction to communication theory). Tome I. Signaux non aléatoires (non random signals). Chap.III. Emploi des distribution (use of distributions), pp.23/52.
if $\tau \in\left[\tau_{1}, \tau_{2}\right] \cap\left[\mathrm{t}_{1}, \mathrm{t}_{2}\right]$, then $\delta[\tau] \bullet \mathrm{g} \prime \prime=\delta[\tau] \bullet \mathrm{g}$
if $\tau \notin\left[\tau_{1}, \square \tau_{2}\right] \cap\left[\mathrm{t}_{1}, \mathrm{t}_{2}\right]$, then $\delta[\tau] \bullet \mathrm{g} \prime \prime=\delta[\tau] \bullet \mathrm{g}_{0}$
The domain of g '" is $\left[\tau_{1}, \tau_{2}\right] \cap\left[\mathrm{t}_{1}, \mathrm{t}_{2}\right]$. This formalism means physically that the rectangular pulse $\delta\left[\tau_{1}, \tau_{2}\right]$ limits the observation of $g$ to the interval [ $\left.\tau_{1}, \tau_{2}\right]$. To simplify the language, we shall also say that either the rectangular pulse breaks up the process $g$ or the process $g$ modulates the rectangular pulse. Note that this formalism is independent of the mathematical nature of g ; it is applicable in particular when g is an analytical function in the form of a simple product of a function multiplied by a distribution.

The simultaneous application of hypotheses 12 and 13 leads the following theorem (Appendix, page 61, dem. 2):
th. 1: if $\tau \in\left[\tau_{1}, \tau_{2}\right] \cap\left[t_{1}, t_{2}\right]$, then $\delta[\tau] \otimes g ’=\delta[\tau] \otimes g$
else
if $\tau \notin\left[\tau_{1}, \boxtimes \tau_{2}\right] \cap\left[\mathrm{t}_{1}, \mathrm{t}_{2}\right]$, then $\delta[\tau] \otimes \mathrm{g}^{\prime}=\delta[\tau] \otimes \mathrm{g}_{0}=\mathrm{a}_{0}$

- Comments

Note should be made of the difference between the two notions previously defined: the simple sample is a process, an element of $G$, and the instantaneous value is the image of this sample in the observation space A.

Example 30: g represents an electric voltage, $\delta[\tau] \bullet \mathrm{g}$ is what is restored by a sampler; $\delta[\tau] \otimes \mathrm{g}$ is what is restored by a converter.
Example 31: g represents the changes in the balance of a bank account, $\delta[\tau] \bullet \mathrm{g}$ is the balance at a given date and time; $\delta[\tau] \otimes \mathrm{g}$ is the message delivered by the visual display of an automatic cash register at the same date and time.

### 2.2.4 Structures in classes

a) Structure based upon process recursiveness

We are concerned with phenomena which are reproducible in time. The hypothesis used in this paragraph is that any phenomenon in the functioning of a system is reproducible.

Let $g$ and $g$ ' be two elements of $G$; $g$ and g' are equivalent if they represent the same phenomenon, to within a translation in time. Let $\rho$ be the equivalence relation based upon this property; the quotient set, the elements of which are the equivalence classes is noted: $\Gamma=\mathrm{G} / \rho$.
hyp. 14: Whatever $g$, there is, for any value of the real number $\theta$, a translated $g$ ' such that:

$$
(\forall \mathrm{t} \in \mathrm{~T}), \delta[\mathrm{t}-\theta] \otimes \mathrm{g}^{\prime}=\delta[\mathrm{t}] \otimes \mathrm{g}
$$

b) Structure based upon the existence of an initial instant

We have associated a closed set $\left[\mathrm{t}_{1}, \mathrm{t}_{2}\right] \subset \mathrm{T}$ as a domain with any existing element $\mathrm{g} \in \mathrm{G}$. This notion of domain must be explained. In fact, for the moment, the choice of the domain appears to be arbitrary: whatever $\left[\mathrm{t}_{1}, \mathrm{t}_{2}\right]$, it is possible to find $\left[\mathrm{t}^{\prime}, \mathrm{t}^{\prime}{ }_{2}\right] \neq\left[\mathrm{t}_{1}, \mathrm{t}_{2}\right]$ where $\left[\mathrm{t}_{1}, \mathrm{t}_{2}\right] \subset\left[\mathrm{t}_{1}, \mathrm{t}^{\prime}{ }_{2}\right]$, and then substitute $\left[\mathrm{t}_{1},,^{\prime}{ }_{2}\right]$ for $\left[\mathrm{t}_{1}, \mathrm{t}_{2}\right]$ without calling into question the stated hypotheses. $\left[\mathrm{t}^{\prime}, \mathrm{t}^{\prime}{ }_{2}\right]$ is thus also a domain. To clear this indetermination, we express the intuitive fact that after $\mathrm{t}_{1}$ "there is something happening" and that, after $\mathrm{t}_{2}$ "nothing is happening anymore".

Let us consider the subset $\mathrm{T}^{\prime}$ of the instants $\mathrm{t}^{\prime}$ of $\left[\mathrm{t}_{1}, \mathrm{t}_{2}\right]$ such that for any $\mathrm{t}^{\prime}, \delta\left[\mathrm{t}^{\prime}\right] \bullet \mathrm{g}=\delta\left[\mathrm{t}^{\prime}\right] \bullet g_{0}$. Each $t$ thus defined is enclosed in at least one open set whose intersection with $\left[t_{1}, t_{2}\right]$ does not contain any other element of T'. Intuitively, the instants such as t' are "isolated" within the domain of the process:
hyp. 15: There is at least one open set $\mathrm{O}_{\mathrm{t}^{\prime}}$ containing $\mathrm{t}^{\prime}$ such that for any $\mathrm{t} \in \mathrm{O}_{\mathrm{t}} \cap\left[\mathrm{t}_{1}, \mathrm{t}_{2}\right]$ and $\mathrm{t} \neq \mathrm{t}^{\prime}$,
$\delta[t] \bullet g \neq \mathrm{g}_{0}$

- Comments
$t_{1}$ and $t_{2}$, the left and right borders of the domain can be elements of $T^{\prime}$.
Example 32: If the process is a variation in electric voltage $u()$, the hypothesis means that $u(t 1)=0$ but that immediately after and for a non-zero duration $(\mathrm{t}) \neq 0$. The hypothesis brings mathematical rigour to physical fuzziness. Any computer specialist having used industrial instrumentation knows that, beyond a certain accuracy threshold, it is difficult to locate the
initial instant of the occurrence of an electric signal. This imprecision was one of the main causes for the abandonment of the "single-slope" analogue converter in benefit of the "dual-slope" converter eliminating this drawback.

Let $g$ and $g$ ' be two elements of $G$ having respectively $\left[\tau_{1}, \tau_{2}\right]$ et $\left[\tau^{\prime}{ }_{1}, \tau^{\prime}{ }_{2}\right]$ as domains; $g$ and $g$ ' are equivalents if and only if their respective domains have the same initial instant which is their common lower bound: $\tau_{1}=\tau^{\prime}{ }_{1}=\tau$; Let us designate as $\rho$ ' the equivalence relation based upon this property; the quotient set is noted as: $\Gamma^{\prime}=\mathrm{F} / \rho^{\prime}$.

## c) Properties common to the two structures

Hereafter we shall use the notation $\Gamma^{\prime}=\left\{\gamma_{t}^{\prime}\right\}$; intuitively $\gamma_{t}^{\prime}$ is the class of all $g$ having the same «sfating time » $\mathrm{t} ; \gamma_{0}^{\prime}$ is in particular the element of $\Gamma^{\prime}$ associated with the instant 0 ; let us note $\gamma_{0}^{\prime}=\{g\}$ where $g$ represents any $g$ having 0 as starting time. By construction also, there is a bijection between any class $\gamma^{\prime}{ }_{t}$ and $\Gamma$; let us choose class $\gamma^{\prime}{ }_{0}$; any g represents a class of $\Gamma$. From these properties, there is obvious theorem.
th. 2: Any $t$ defines an operator $\Theta_{t}$ in $\Gamma^{\prime}$ (Annexe, Page 61, dem. 1): $\Theta_{\mathrm{t}}\left(\gamma_{\tau}{ }_{\tau}\right)=\gamma^{\prime}{ }_{\mathrm{t}-\mathrm{\tau}}$.

### 2.3 Substrate space

### 2.3.1 <br> Purpose

A process is a modelling element; it defines a structured collection of processes defined in time except for a translation; the study of the general properties of this element is based upon the definition of the substrate space.

### 2.3.2 <br> Definitions

def. 4: Any class, element of $\Gamma^{\prime}$, constitutes a substrate element.
def. 5: Conventionally, we choose the class associated with the instant 0 . This class is the substrate space.

In the rest of the document we are using the notations:
$\{g\}=q=\gamma_{0}^{\prime} \in \Gamma^{\prime}$.
Class $\mathscr{G}=\gamma^{\prime}{ }_{0}$ is included in $G$, and the hypothesis relative to the observation (hyp. 13) is applicable; it implies the existence of an application o: $\Delta \times \mathfrak{q} \rightarrow \boldsymbol{q}$.

For any $\delta\left[\tau_{1}, \tau_{2}\right] \in \Delta$ and any $g \in \mathscr{G}$ having a domain $\left[0, \mathrm{t}_{2}\right]$, there is $\mathrm{g} \boldsymbol{} \in \mathrm{G}$ such that:
if $\tau \in\left[\tau_{1}, \tau_{2}\right] \cap\left[0, \mathrm{t}_{2}\right]$ then $\delta\left[\tau-\tau_{1}\right] \otimes \mathrm{g}^{\prime}=\delta[\tau] \otimes\left(\delta\left[\tau_{1}, \tau_{2}\right] \bullet q\right)=\delta[\tau] \otimes q$
if $\tau \notin\left[\tau_{1}, \tau_{2}\right] \cap\left[0, \mathrm{t}_{2}\right]$ then $\delta\left[\tau-\tau_{1}\right] \otimes \mathrm{g}^{\prime}=\delta[\tau] \otimes\left(\delta\left[\tau_{1}, \tau_{2}\right] \bullet g\right)=\mathrm{a}_{0}$

The left bound of the domain of $g^{\prime}$ is $\tau_{1}$; then $g$ ' is an element of the class $\gamma^{\prime}{ }_{\tau_{1}}$; $\tau_{1}$ defines an operator $\Theta \tau_{1}$ in $\Gamma^{\prime}$ such that: $\Theta \tau_{1}\left(\gamma^{\prime}{ }_{\tau_{1}}\right)=\gamma^{\prime}=\underline{g}$. Consequently, $g^{\prime}$ has an image $g^{\prime}$ in $g$.
th. 3: If $g$ is a process, then $g$ ' its observation,

- obtained during an interval $\left[\tau_{1}, \tau_{2}\right]$ and,
- translated to the origin, is also an element of $g$ :
$g^{\prime}=\delta\left[\tau_{1}, \tau_{2}\right]$ o $g \in \mathcal{G}$


### 2.3.3

## Properties of substrate space

The following hypotheses and definitions are given for the substrate space $q$ conventionally associated with the instant 0 . They are immediately applicable to any translated substrate space of the preceding and associated with any other instant $t$. Let us consider: $\mathrm{P}(\underline{q})$, the set of subsets of $\mathfrak{q}$.

## a) Family of states

def. 6: A substrate state $g$ is a part $X \in \mathcal{P}(\mathscr{G})$


Figure 14: Interpretation of notion of state
def. 7: $P_{g}(G)$ is the family of the substrate
states associated with $g$ : for any $X \in \mathcal{P}_{g}(\underline{g}), g \in X$,
The following two hypotheses enable us to assign a topological structure to the substrate space.

## - Comments

This topological structure is not unique: other structures depending on the properties of the elements may be introduced later. Depending on the particular physical properties of the represented entity, they are not examined in this paragraph.

## b) Convergence

hyp. 16: For any $X$ and $X^{\prime} \in \operatorname{Pg}(\mathcal{G})$,
there is an $X$ '" such that:

$$
X^{\prime \prime} \subset X \cap X^{\prime} \text { and } X^{\prime \prime} \in p_{g}(g) .
$$

The intersection of two substrate states, elements of a given family, contain another substrate state belonging to this family.

## - Comments



Figure 15: State families - convergence

This hypothesis again states with rigour an idealised physical fact.

Example 33: Let us suppose that the same process $\mathfrak{g}$ is observed with several carefully adjusted and calibrated observation systems all giving an image of $g$ in A. For example, the preceding electric voltage $u()$ measured with several voltmeters. Each image is a bundle in $\mathrm{A} \times \mathrm{T}$. The above hypothesis means that all these bundles have a non-empty intersection; this intersection can contain another bundle possibly produced by another observation system of better precision than the preceding ones; the perfect image of $q$, indeterminable with full certainty, is somewhere within this latter bundle.
c) Neighbourhood
hyp. 17: For any $\mathrm{X} \in \boldsymbol{P} g(\mathcal{G})$ and for any $h \in \mathrm{X}$, there is $\mathrm{Y} \subset \mathrm{X}$ with $g \in \mathrm{Y}$ and $\mathrm{Y} \in \mathcal{P} h(\mathcal{q})$


Figure 16: Family of states - continuity

The collection of all the families defines a topology for $\mathfrak{g}$.

## d) Primitive element definition limits

The belonging of $\{g\}$ to $P_{g}(g)$ will be discussed later; $g$ is a limit element; $\{g\}$ is a limit state.. This is a hypothesis which, depending on whether it is accepted or rejected, modifies the topological properties of the substrate space. Intuitively, the hypothesis will be admitted for the construction of models of real phenomena perceived through observation systems which yield only imperfect images. Rejection will be used for the construction of models of phenomena which are well controlled in their occurrences, such as executions of numerical models; in this case, the observation system could communicate an exact trace of the occurrence.

## 3 <br> RECURSIVE ELEMENTS

### 3.1 Method and constructional principles

The theoretical elements examined earlier are used in this chapter for the construction of system models. Two approaches are possible.


Figure 17: Analysis of an entity

- produces,
- devices.


## B) Synthesis

Given a collection of entities, synthesis consists in assembling these entities, possibly modified, and explaining their interactions. We examine how theoretical elements are used to form the morphology and then the physiology of the new entity thus constructed.

These approaches are covered in detail in the following paragraphs.

### 3.2 Elements for morphological design

### 3.2.1

## General

There are two possibilities for observing the functioning of a physical entity:

- There is an observation system allowing a correspondence between any instant and an instantaneous value in the observation space. The entity is then considered to be a produce
- There is an observation system enabling the preceding procedure for produces placed at the input of the physical entity as well as for produces coming from its outputs. The entity is then considered to be a device.

A procedure is a model describing the general functioning rules of a produce or a device; a procedure instantiation is a process. The notion of


Figure 18: Two possibilities for observing and two objectives of modelling component having different possible states is almost universal. It appears at different moments of an analysis: a variable, a task, a program, the system itself are all elements having this common physical property, independent of any other property. These two possibilities for observing imply two ways for modelling. The theory of categories brings the required formal means to the representation of these basic properties

### 3.2.2

## Produce model

## a) Definitions

Given a family of processes, only some open sets X (see Fig. 14, Page 30) are of interest for the study of this family. Consequently, we are assuming here that there is at least one condition $\Phi$ available for defining a particular subset: $S=\Phi(\mathcal{P}(\mathcal{G})) \subset \mathcal{P}(\mathcal{G})$.
def. 8: Each element of $S=\Phi(\mathcal{P}(\mathcal{G}) \subset \mathcal{P}(\mathcal{G})$ is a procedure state, chosen according to the condition:
$\Phi: S=\{X / X \in \mathcal{P}(\mathscr{q})$ and $\Phi(\mathrm{X})=$ true $\}$
def. 9: A limit state is an open set in which each element differs very little from a particular element g called the limit element or is reduced to this element.

Let us note that it is experimentally impossible to distinguish a limit element. Any means of observation gives only knowledge of a limit state, the scattering of this limit state depending on the accuracy of the observation means used. However, at this level of analysis, it is impossible, as we have already noted, to affirm or invalidate the belonging of g to S .

The theory of categories is used with the remark that there are relations between the states assigned to a procedure. Let us now consider a set of such states associated with a procedure. We assume that:
hyp.18: For any pair ( $X_{\alpha}, X_{\beta}$ ) of states there is a set $M_{\alpha \beta}$ of relations from $X_{\alpha}$ to $X_{\beta}$; if ${ }_{\alpha}={ }_{\beta}$, this set is reduced to the singleton formed by the identical relation; each relation is called a morphism; For any $\left(X^{\prime}{ }_{\alpha}, X^{\prime}{ }_{\beta}\right), M_{\alpha \beta}=M^{\prime}{ }_{\alpha \beta}$ implies that $X^{\prime}{ }_{\alpha}=X_{\alpha}$ and $X^{\prime}{ }_{\beta}=X_{\beta}$. In this study, any $M_{\alpha \beta}$ is, either empty, or a singleton having the element $m_{\alpha \beta}$.
hyp. 19: For any triplet $\left(X_{\alpha}, X_{\beta}, X_{\chi}\right)$ such as $\mathrm{M}_{\alpha \beta} \neq \varnothing$ and $\mathrm{M}_{\beta \chi} \neq \varnothing$ and $\mathrm{M}_{\alpha \chi} \neq \varnothing$, there is a relation:
$\mathrm{m}_{\alpha \beta} \times \mathrm{m}_{\beta \chi} \rightarrow \mathrm{m}_{\alpha \chi}$
hyp. 20: The relation defined above is associative:
$\left(m_{\alpha \beta} \times m_{\beta \chi}\right) \times m_{\chi \delta}=m_{\alpha \chi} \times m_{\chi \delta}=m_{\alpha \beta} \times\left(m_{\beta \chi} \times m_{\chi \delta}\right)=m_{\alpha \beta} \times m_{\beta \delta}$


Figure 19: Approximative illustration of the relationships between physical concepts and theoretical elements (morphology)

These hypotheses concerning the properties of procedures are such that the notion of category is utilizable as models for the states of procedures. The states are the objects of the category and the state changes are morphisms (Figure 19); ordinarily, the sets of morphisms in this case of application have at most an element.

## b) Discussion on the continuity of states

th. 4: The substrate space has the power of the continuous (Appendix, Page 61, dem. 3.).
The set of states has a power. This power depends on the definition of S. The set of states which is a set of open subsets of the substrate space, may be either continuous or countable.

## c) Combination of produces

Let be a functionality $F$ having several inputs $I_{i}$, $\mathrm{i}=1, \ldots, \mathrm{n}$. If each $\mathrm{I}_{\mathrm{i}}$ is a category, then the model of the compound produce $I_{1} \times \ldots I_{n}$ is a product of categories (see: appendix).

We will use this property in the following section.

### 3.2.3

Device model


Figure 20: Compound produce

We shall examine first of all a simple functionality giving a transformation of a produce at the input into an output produce. We represent the input (respectively the output) by an input category (respectively an output category). We then assume that the functionality applies the objects of the input category (respectively the morphisms) in the objects of the output category (respectively the morphisms).
hyp.21: We assume that the application carried out by the functionality complies with the axioms defining a fonctor.
A) Simple one-state device
def. 10: The simple one-state device embeds a functionality defining a correspondence between the inputs and the outputs.
The model of each input or of each output is a product of categories. The model of the functionality is a fonctor; therewith the model of the simple one-state device is a fonctor.

## b) Compound one-state devices

We examine two simple compositions of simple one-state devices. The result is a device which embeds:

- either a serial assemblage of one state-devices
- either a parallel assemblage of one-state devices.

A collection of functionalities $F_{1}, \ldots, F_{n}$ is a serial collection if the input $I_{i}$ of any $F_{i}(n \geq i>1)$ is the output $\mathrm{O}_{\mathrm{i}-1}$ of $\mathrm{F}_{\mathrm{i}-1}$. The result is a functionality such that: $\mathrm{F}: \mathrm{I}_{1} \rightarrow \mathrm{O}_{\mathrm{n}}$.
def.11: The serial compound one-state device is a device which embeds a serial collection of functionalities.

The model of this device is the fonctor $\mathrm{F}: \mathrm{I}_{1} \rightarrow \mathrm{O}_{\mathrm{n}}$.
A collection of functionalities $\mathrm{F}_{1}, \ldots, \mathrm{~F}_{\mathrm{n}}$ is a parallel collection if any pair ( $\mathrm{F}_{\mathrm{i}}, \mathrm{F}_{\mathrm{j}}$ ) of this collection exists without link ${ }^{30}$; The functionality $\mathrm{F}:\left(\mathrm{I}_{1} \times \ldots \mathrm{I}_{\mathrm{n}}\right) \rightarrow\left(\mathrm{O}_{1} \times \ldots \mathrm{O}_{\mathrm{n}}\right)$ represents this collection.
def.12: The parallel compound one-state device is a device which embeds a parallel collection of functionalities.
The model of this device is the fonctor $\mathrm{F}:\left(\mathrm{I}_{1} \times \ldots \mathrm{I}_{\mathrm{n}}\right) \rightarrow\left(\mathrm{O}_{1} \times \ldots \mathrm{O}_{\mathrm{n}}\right)$.

## c) Multi-states device

A functionality may be a product having multiple states. This assumption is very usual in data processing technologies. In this case, each state of the functioality may be viewed as a simple functionality. We examine now the features of a device embeding such a functionality F. F may be viewed as


Figure 21: multiple states functionality a fonctor or as a caregory if we need to take in account its different states. Let be:

- $\mathrm{F}=\left(\left\{\mathrm{F}_{\mathrm{i}}\right\},\left\{\mathrm{f}_{\mathrm{j} i}\right\}\right)$, the category, model of the multi-states functionality; each Fi is a fonctor and represents a simple-state functionality.
- $\mathrm{C}=\left(\left\{\mathrm{C}_{\mathrm{i}}\right\},\left\{\mathrm{c}_{\mathrm{ji}}\right\}\right)$, a category, model of the input data used in order to control the states of F
- $\mathrm{I}=\left(\left\{\mathrm{I}_{\mathrm{a}}\right\},\left\{\mathrm{m}_{\mathrm{ba}}\right\}\right)$, a category, model of the input of the device.
- $\mathrm{O}^{\prime}=\left(\left\{\mathrm{O}^{\prime}{ }_{(\mathrm{ia})}\right\},\left\{\mathrm{n}^{\prime}{ }_{(\mathrm{jb})(\mathrm{ia})}\right\}\right)$, a category, model of the output of the device.

We assume that:
hyp.22: it exists an one to one application between $\left\{\mathrm{C}_{\mathrm{i}}\right\}$ and $\left\{\mathrm{F}_{\mathrm{i}}\right\}$,
hyp. 23: it exists an one to one application between $\left\{\mathrm{c}_{\mathrm{ji}}\right\}$ and $\left\{\mathrm{f}_{\mathrm{ji}}\right\}$,
hyp.24: O ' is a subcategory of a category O which has the following properties;
hyp. 25: For any i and $\mathrm{a},\left(\mathrm{C}_{\mathrm{i}}, \mathrm{I}_{\mathrm{a}}\right) \rightarrow \mathrm{O}_{\mathrm{i} a}=\mathrm{F}_{\mathrm{i}}\left(\mathrm{I}_{\mathrm{a}}\right)$;
hyp. 26: For any (j,i) and (b,a),
$\left(\mathrm{c}_{\mathrm{ji}}, \mathrm{m}_{\mathrm{ba}}\right) \rightarrow \mathrm{n}_{\mathrm{ji})(\mathrm{ba})}$
and
$\mathrm{O}_{\mathrm{jb}} \quad=\mathrm{n}_{(\mathrm{ji})(\mathrm{ba})} \mathrm{O}_{\mathrm{i} \mathrm{a}}$
$=\left(\mathrm{c}_{\mathrm{ji}}, \mathrm{m}_{\mathrm{ba}}\right)\left(\mathrm{F}_{\mathrm{i}}\left(\mathrm{I}_{\mathrm{a}}\right)\right)$
$=\left(\mathrm{c}_{\mathrm{ji}} \mathrm{F}_{\mathrm{i}}\right)\left(\mathrm{m}_{\mathrm{ba}} \mathrm{I}_{\mathrm{a}}\right)$.
All these hypotheses allow the existence of a fonctor G : $\mathrm{C} \times \mathrm{I} \rightarrow \mathrm{O}$ '. This fonctor is the model of the device embeding the multi-state functionality. Globaly, this device is a one-state functionality having the product $\mathrm{C} \times \mathrm{I}$ as input and $\mathrm{O}^{\prime}$ as output.

Let us note that the last hypothesis (26) is not a universal property of the product of categories; this hypothesis is restrictive for modelling any component of a system. At this point we can imagine, as we like, some multi-state component losing the properties of fonctor. Then, the result is not a device; all the components which embed recursively this item are also losing the properties of a fonctor and are not devices. Whe have not been studying at this time the effects of such an arrangement; these effects transform certainly the conditions of transportability and of interoperability and complicate their specifications. For that reason, the existence of softwares available for checking in a system description the different possible lacks of fonctor properties.


Figuree 22: Approximative illustration of the relationships between physical concepts and theoretical elements (morphology)

## D) Special types of devices: The receiver, transmitter, connector

The model of the receiver, transmitter or connector is a procedure. This procedure is also a fonctor; it specifies the criteria for the recognition of categories or generation of categories at the border of a procedure. These are fonctors to the extent that generally they impose transformations on the categories mentioned in the section on the borders of the procedure.

Example 34: A receiver receives a pulse train but, in each pulse, recognises only the rising edge. A transmitter transforms a pulse train generated inside the procedure into a continuous signal whose frequency is that of the occurrences of the distinct pulses.

### 3.2.4 Representation of fundamental properties

The category serves as a model for the produces; the fonctor serves as a model for the devices.

## A) Existence

A fonctor which establishes a relation between a product of categories and the category (false, true).

## B) Temporal domain

For any category C and for any category limited to a single object $\theta \in \mathrm{T}$, an operator defines a transformation: C $\rightarrow \mathrm{C}^{\prime} \times \theta$

Example 35: A sampler defines a category C' having the same properties as C over the interval $\theta$.
An operator is fundamentally the base of a rule of meta production indispensable in any system description language:
operator: morphology $\rightarrow \square$ morphology
In a description, the operator allows a simple definition of a produce based on a type given by modification of the domain.

## C) Linkage

A fonctor intended to identify the links between I/Os of different procedures.
Example 36: The "identical" fonctor used to specify that the inputs of one procedure are the outputs of another procedure and are identical.

## $3.3 \quad$ Elements for physiological design

### 3.3.1

General

The analysis of a system leads to the identification of its components. These components - devices and produces - form the morphology of the system; this morphology evolves as the analysis progresses. The notion of category serves as a model for the produces. Analysis determines the recognition of devices and produces meeting the general characteristics of Figure 17. Modelling should make it possible, for each produce, to specify its different possible states. It must allow, for each device, an account:

- of the produces exchanged and of the morphological specifications imposed by the device on these produces, possibly specifications relative to processing performed on these produces,
- of its location in time.


Figure 23: Approximative illustration of the relationships between physical concepts and theoretical elements (physiology)

### 3.3.2

 Command or Timing-operatorA command is a fundamental element of the meta-production rule:
command: morphology×time $\rightarrow$ physiology
This meta-production rule has two physical acceptations described by the following mathematical models. The behaviour of a system is the definition of a temporal location of each active part of the system. Any fonctor used in the definition of a behaviour transforms a given category C (see the Section «Produce model», Page 32) into a other category C' taking in account its location in time. This kind of fonctor is a command or a timing-operator. For any category C, two type of command are available.

## A) On/off command

Let be $\mathcal{P}(\mathrm{T})$ the set of the open subsets of $\mathrm{T}: \mathcal{P}(\mathrm{T})=\{ ] \tau_{\mathrm{i}}, \tau_{\mathrm{j}}[ \}$ and $\Theta$ the n-uple such that:
$\Theta=(] \tau_{\mathrm{i} 1}, \tau_{\mathrm{j} 1}[, \ldots,] \tau_{\mathrm{in}}, \tau_{\mathrm{jn}}[) \in \boldsymbol{P}^{\prime}(\mathrm{T})$
with the condition: for any $\alpha$ and $\beta, \alpha \neq \beta, 1 \leq \alpha \leq n, 1 \leq \beta \leq n,] \tau_{\mathrm{i} \alpha}, \tau_{\mathrm{j} \alpha}[\cap] \tau_{\mathrm{i} \beta}, \tau_{\mathrm{j} \beta}[=\varnothing$.


Figure 24 : ON/OFF command
$\Theta$ is a category; it models a sequence of disjointed windows along the time-scale; it has n objects: the intervals $] \tau_{\mathrm{i} \alpha}, \tau_{\mathrm{j} \alpha}[(1 \leq \alpha \leq \mathrm{n})$; these objects are ordered and this relationship defines a morphism for each pair of objects.

For any category C which models a static object, and for any category $\Theta$ which is the support of the command, an On/Off command defines the produce: $\mathrm{C}^{\prime}=\mathrm{C} \times \Theta$. Each object of this category is a pair $\mathrm{X}^{\prime}{ }_{u}$ $=\left(\mathrm{X}_{\mathrm{u}},\right] \tau_{\mathrm{i} \alpha}, \tau_{\mathrm{j} \alpha}[)$; this pair means that :

- each element $g^{\prime}$ of $X_{u}^{\prime}$ is the translation in time, from 0 to $\tau_{i \alpha}$ of an object $g$ of $X_{u}$,
- the left bound of the domain of each $g$, element of $X_{u}$ is $\tau_{i \alpha}$, and that each $g$ is sampled after this translation in time.


## B) Trigger or pulse-command

Let be $\boldsymbol{P}^{\prime}(\mathrm{T})$ the set of the open subsets of $\mathrm{T}: \mathcal{P}(\mathrm{T})=\{ ] \tau_{\mathrm{i}} \pm \varepsilon[ \}$ and $\Theta^{\prime}$ the n -uple such that:
$\Theta^{\prime}=(] \tau_{1} \pm \varepsilon[, \ldots,] \tau_{n} \pm \varepsilon[) \in \mathcal{P}^{\prime}(T)$ with the condition: for any i and $\mathrm{j}, \mathrm{i} \neq \mathrm{j}, 1 \leq \mathrm{i} \leq \mathrm{n}, 1 \leq \mathrm{j} \leq \mathrm{n}, \tau_{\mathrm{i}} \neq \tau_{\mathrm{j}}$. $\Theta^{\prime}$ is a category; it has $n$ objects: the intervals $] \tau_{\mathrm{i}} \pm \varepsilon$ [ $(1 \leq \mathrm{i} \leq \mathrm{n})$; these objects are ordered and this relationship defines a morphism for each pair of objects.

The trigger transforms $C$ into $C^{\prime}=C \times \Theta^{\prime}$. Each object of this category is a pair $X^{\prime}{ }_{u}=\left(X_{u},\right] \tau_{\mathrm{i}} \pm \varepsilon$ [); this pair means that:

- each element $g^{\prime}$ of $X_{u}^{\prime}$ is the translation in time, from 0 to $\tau_{i}$ of an untimed element $g$ of $X_{u}$,
- the left bound of the domain of each $g$ ', element of $X_{u}$ is $\tau_{i}$, and $g$ ' is equal to $g$ after this translation without sampling.


Figure 25 : Trigger command

### 3.3.3 Temporal condition

Let us note that any command can be combined with a condition using T .

Example 37: Activate a procedure at an instant t , generate a pulse at an instant t .

## INTEROPERABILITY

The models of entities are assemblies of the previously described theoretical elements. The properties of these assemblies result from the hypotheses and definitions proposed earlier. We shall be examining more particularly in this chapter the ability of the assembled entities to function interactively: they indicate the physical consistency of the system.

The mathematical model enables this property to be studied by offering the means of distinguishing its different aspects. We shall thus be examining in particular the questions of interoperability and portability, linking them with the examination of the consistency of the mathematical expressions of the entities concerned.

As we stated earlier, interoperability is the ability of a procedure $\mathbf{F}$ ' to accept any produce sent by another procedure $\mathbf{F}$. Let $\mathbf{O}$ be the category sent by $\mathbf{F}$, and $\mathbf{I}$ ' the category that can be received by $\mathbf{F}^{\prime}$. The consistency between $\mathbf{O}$ and $\mathbf{I}^{\prime}$ exhibits different aspects. We shall attempt now to classify, to define some consistency criteria between procedures acting interactively.

## 4.1

## Morphological consistency

This notion is intrinsic to the pair ( $\mathbf{F}, \mathbf{F}^{\prime}$ ). Its modelling uses the mathematical concept of universal element (Appendix, Page 60). Let $\mathbf{I}^{\circ}$ be a subcategory of $\mathbf{I} ’$; we propose:
def. 13: Normal consistency. F and $\mathbf{F}$ ' are normally consistent for $\mathbf{O}$ and $\mathbf{I}^{\prime}$ if there is an abstract fonctor $\Gamma: \mathbf{O} \rightarrow \mathbf{I}^{\circ}$ having at least one universal element. $\Gamma$ defines a standard interface.
def. 14: Adapted consistency $\mathbf{F}$ and $\mathbf{F}$ ' are adapted for $\mathbf{O}$ and $\mathbf{I}^{\prime}$ if there is a fonctor $\Gamma^{*}$ : $\mathbf{I}^{*} \rightarrow \mathbf{O}^{* \circ}$ having at least one universal element such that ( $\mathbf{F}, \Gamma^{*}$ ) and ( $\Gamma^{*}, \mathbf{F}^{\prime}$ ) are normally consistent. $\Gamma^{*}$ defines an adapted interface.

## 4.2

 Physiological consistencyThis notion is concerned fundamentally with real time. It is intrinsic to the pair ( $\mathbf{F}, \mathbf{F}$ ) but introduces time. Let us consider a closed set [ $\tau_{1}, \tau_{2}$ ]; the set of open subsets included in [ $\tau_{1}, \tau_{2}$ ] forms a category. Since real time is physically invariant in our applications, every fonctor $\Gamma$ transforms this category in itself and conserves its morphisms. There is a universal element in this case which is [ $\tau_{1}, \tau_{2}$ ]. For any $\tau$, let us consider the instantaneous open set $[\tau \pm \varepsilon]$ and $\mathrm{f}:\left[\tau_{1}, \tau_{2}\right] \rightarrow[\tau \pm \varepsilon]$; for any $\left[\tau^{\prime} \pm \varepsilon\right]$ there is a unique f such that: f ( $\left[\tau_{1}\right.$, $\left.\left.\tau_{2}\right]\right)=\left[\tau^{\prime} \pm \varepsilon\right]$. If $\mathbf{F}$ and $\mathbf{F}^{\prime}$ are naturally consistent, f is such that $[\tau \pm \varepsilon]=\left[\tau^{\prime} \pm \varepsilon\right]$; if $\mathbf{F}$ and $\mathbf{F}$ ' are not naturally consistent, f depends on the artificial interface.

It is now possible that $\mathbf{F}$ and $\mathbf{F}$ ' are active in the different intervals: $\left[\tau_{1}, \tau_{2}\right],\left[\tau^{\prime}{ }_{1}, \tau^{\prime}{ }_{2}\right]$.
def.15: Physiological interoperability exists if:

- $\mathbf{F}$ and $\mathbf{F}$ ' are consistent,
- $\left[\tau^{\prime} \pm \varepsilon\right] \subset\left[\tau^{\prime}{ }_{1}, \tau^{\prime}{ }_{2}\right]$.


## Portability

The notion of portability can be expressed in the theory of categories. In the following proposition we use the mathematical notion of natural transformation (Appendix, Page 59). An even superficial examination of portability shows the complexity of this notion, owing to the existence of what we conventionally call the service layers. The following proposition is thus only a theoretical element applicable to a physical system element; this theoretical element is thus available for the construction of a complete model specific to each case examined; it is not a universal model.

Let us consider two platforms " m " and " n "; we shall assume in our proposal that the binary element configurations used to store information have the same number of elements in m and in n and that m and $n$ have the same number of such configurations ${ }^{31}$. Let us consider an application (computer) $\mathbf{S}$ and its implementation $\mathbf{S}_{\mathbf{m}}$ in the platform $m$ (resp. $\mathbf{S}_{\mathbf{n}}$ in n ). $\mathbf{S}_{\mathbf{m}}$ and $\mathbf{S}_{\mathbf{n}}$ are fonctors. The common input of these implementations is a category $\mathbf{C}$ situated at the level of the user (for example, a record on a user's data disk). We thus assume here that $\mathbf{S}_{\mathbf{m}}$ and $\mathbf{S}_{\mathbf{n}}$ include the components allowing the implementation of C in the different platforms. The outputs $\mathbf{S}_{\mathbf{m}}(\mathbf{C})$ and $\mathbf{S}_{\mathbf{n}}(\mathbf{C})$ are respectively in accordance with the specifications of platforms which support the applications. If C' is the category formed by the ordered set of all the possible combinations of valorised binary element configurations of $m$ or $n, \mathbf{S}_{\mathbf{m}}(\mathbf{C})$ and $\mathbf{S}_{\mathbf{n}}(\mathbf{C})$ are subcategories generally different from $\mathbf{C}$,

Using the notion de natural transformation, we express the portability of $S$ from $m$ to $n$ by the existence of a relation between the installed applications $\mathbf{S}_{\mathbf{m}}$ and $\mathbf{S}_{\mathbf{n}}$, leading to a relation between the outputs $\mathbf{S}_{\mathbf{m}}(\mathbf{C})$ and $\mathbf{S}_{\mathbf{n}}(\mathbf{C})$ for any input $\mathbf{C}$.

[^4]

Fig. 26: Portability
$\tau: \mathbf{S}_{\mathbf{m}} \rightarrow \mathbf{S}_{\mathbf{n}}$.

def. 16: The above diagram defines an oriented elementary portability from $S_{m}$ to $S_{n}$ (go only).
th. 5: In order for the elementary portability to be reversible between $S_{m}$ and $S_{n}$, it is necessary and sufficient for all the morphisms $\tau$ to be invertibles (go and return).

Conclusion

This part concerned the use of mathematics for system modelling: we have given an overview of some simple physical evidences as we perceive them, and we have consequently proposed their mathematical representations as we imagine them.

In any other usual scientific or technical domain of applied mathematics, anyone, who has been studying about mathematical modelling elements, will have been fascinated by the power of these tools, for discerning and enlightening the properties of the examined physical objects; furthermore these tools avail the growth of design techniques for engineering which gain gradually a consensus from the users and finally lead common practices for technicians who are not mathematicians. An example is given by the finite elements method which was a subject for mathematician at the end of the years fifty and is nowadays an usual tool in all engineering offices and lays the foundation of many civil engineering standards.

The various processes whereby these common practices emerge and evolve are complex. We recognise entirely their end results without explanation. The conclusion is that we must keep in mind the existence of many theoretical researchs undertaken by teams in numerous laboratories and we suggest finally to take them into account in our standardisation activities. We have established the possibility of associating a model with a physical system as well as with each entity recognised in its composition. This model is an assembly of morphological and physiological assertions; such an assembly may comprise several thousand assertions.

The value of mathematics, namely that it produces models which are condensed in terms of expression but rich in terms of meaning, thus appears to be absent from this approach. Reasoning with a model
having such a number of relations is humanly impossible: for example, the simple search for contradictions which could mean potential functioning errors is excluded.

Such formal verifications by reasoning become possible with the assistance of the computer. Classical notations are unfortunately inappropriate. The search for a formal, computer-readable language whose elements and assemblies have the same semantics as the usual mathematical language is thus required. This search can be based upon existing description languages.

It forms the subject of the next part.

## Part 4 DESCRIPTION LANGUAGE (IRON FRAME)

> Toute interdisciplinarité réelle passe par l'édification d'un langage commun qui puisse exprimer les divers moyens théoriques en usage dans les disciplines les plus variées.
> René Thom ${ }^{32}$. Logos Phoenix, Modèles mathématiques de la morphogénèse, Bourgoi, Paris, 1980, p. 295.

During the eighteenth century the existence of such an universal scientific language was soon supposed by Leibnitz. Unfortunately, Leibnitz papers tell us almost nothing about that. It is one of the most famous enigma of mathematics history. We can say nowadays that the binary formalism discovered or re-discovered by Leibnitz is perhaps the basis of this hypothetical language. The conclusion to which we are coming since the beginning of computers era is that in the present state of the technology, binary language is obviously at the lowest level, the only universal language with which any problem of calculation must be expressed and solved; nevertheless, in practice, it is easily readable and understandable only by computer and never by human mind.

While we have this language in view, and while we must leave for the present the study of the conditions under which it may be designed, we should just consider for a moment what we mean by the concept of global description.

At the present day no known high level language, that means readable by computer and undersantable by human mind, possesses without restriction the property of universality. The theory of category underlies the precedent definitions. The theory of category is an algebra; thus it is a logical theory and the axioms, definitions and theorems of fundamental logic are available and therefore must be applied.

A description language is one of the most important items of a CSMF. Consequently, the CSMF standard must provide for these languages:

- A general primary formalism allowing an unitary basic structure.
- General specifications for their capabilities.
- Criteria for checking the compliance of the features of a particular language with the preceding specifications.

The purpose of this chapter is to study the problems concerning the description languages. The first paragraph proposes a simple primary formalism for expressing, at the lowest level, the concepts of CSMF standard and their possible assemblages. The second paragraph examines how that language may be used as a meta-language for producing description languages. In compliance with the primary formalism, the third paragraph gives in natural language some general specifications for the description languages.

## 2.1 <br> Objectives

The purpose of this paragraph is the definition of a primary formalism without ambiguity. This formalism must be viewed firstly as a language a the lowest level ; consequently, it must have only a reduced set of notions and a reduced set of rules. Then, this primitive formalism will be used uniquely for expressing strictly the physical concepts defined informally in the preceding chapter and the relationships between these concepts. It must be consistent with the mathematical elements defined in the next chapter.

This low level formalism cannot be used for the description of systems. Hereafter, the notions and rules of this low level formalism will be used, for that purpose, as a frame for defining description languages. Then, it may be viewed also as a general and formal specification of description languages.

## $2.2 \quad$ General features

This formalism is a direct application of set theory. It uses the habitual notations of this theory:

- braces enclose a repeated item; the item may appear zero or more times; each occurrence of the repeated item is distinct from the other occurrences.
- $\cap, \cup, \times$ represent respectively intersection, reunion and scalar product; $\Pi$ represents a scalar product having an undetermined finite list of terms.
- $\varnothing$ is the empty set.
- The chain,
«<expression_1>: <expression_2> $\rightarrow$ <expression- 3>»
means:
<expression_1> represents an application from
<expression_2>
into
<expression_3>.
- In addition in any definition, the symbol «: := » means «is», and the symbol «--» precedes a informal comment enclosed inside the formal text.
From a mathematical point of view, \{item\} represents the classical set of element «item» which is a list of sentences in the spirit of this chapter.


### 2.3 Primary notions

The terms expressed below are the primitive terms of the formalism. They express both:

- either a concept;
- either a set of sentences associated with the concept, whose elements complies with the concept.
For each word, the existence of the expressed concept is an axiom of the primary formalism. Likewise the existence of the concept, which is expressed by the word « point », is an axiom of geometry. Besides, each word, from (1) to (8), represents any possible element of a set, and likewise the word « point» represents the elements of a geometrical space.
(1) morphological_item::=
(2) morphological_typology _template|
(3) morphological_genealogy _ancestor|
(4) elementary_object
(5) physiological_item::=
(6) physiological_typology _template
(7) physiological_genealogy _ancestor|
(8) action_specification
(9) morphology::= \{morphological_item\}| morphology $\cap m o r p h o l o g y$ morphology $\cup m o r p h o l o g y ~$ morphology $\times$ morphology -- Figure 28, Page 46
(10) physiology::= \{physiological_item\}| physiology $\cap$ physiology| physiology $\cup$ physiology| physiology×physiology -- Figure 29, Page 46
(11) visible_morphology::= morphology
(12)internal_morphology::= morphology
(13) normal_physiology::= physiology
(14) exceptional_physiology::= physiology
(15) object::= morphological_item
visible_morphology Uinternal_morphology Unormal_physiology
Vexceptional_physiology


## 2.5 <br> Primary rules

(16) time::= object
(17) morphological_production_rule::= morphology | operator: morphology $\rightarrow$ morphology
(18) physiological_production_rule::= command: morphology×time $\rightarrow$ physiology


Figure 28: Map of morphological concepts
Figure 29: Map of physiological concepts

## 3

A description language is a formal tool which can actually express systems and must conform with a specification.

To express something in this field of activity means to ship description of physical system between human cognition and computer artificial mind. A computer needs an immuable rigor at any level of information, within the expression of any system, in order to well scan this expression. Yet, the human cognition involves, some other supplementary features for describing its knowledge of a system or well acquiring it; for example the possibility to emphasise away in time or place some global aspects of a description without erosion for the others, particularly those abundant of the deepest level of detail.

Consequently, there is an outline of a set of specifications for defining system description language. All these specifications are consistent with the frame given by the primitive formalism: this mathemùatical origin is a garantee for the rigor. Some others introduce the features wich are necessary for the ease of human mind, without change for the prevalent rigor.

Spec. 1 High level descriptive language with semantic and syntax based upon a sound abstract formalism: the descriptions using this language may be processed for formal validation.
Spec. 2 The language must allow global descriptions of objects belonging to different subject areas (multidisciplinarity). The concept of global description is defined in the formalism.

## Comment on

Spec. 2

Spec. 3 The language must allow the description of collection of objects
Spec. $4 \quad$ The language must allow the description of a set of objects.
Comment on
Spec. 4

Spec. 5
Spec. 6

Spec. 7 For any given object, in assistance of a top-down analysis, the language must allow the expression of the collection of the lower level objects which compose this given object (analysis of an object morphology).
Comment on Spec. 7

Spec. 8 For any given collection of objects, in assistance of a bottom-up analysis, the language must allow the description of any upper-level object having this collection as morphology (synthesis of an object morphology).

Comment on The synthesis of an object is obtained whenever other objects are gathered in order to form its morphology. It is then Spec. 8 necessary to examine how the combination of these objects can act. Two cases are possible:

- The definitions of the lower level object behaviours are sufficient to ensure a complete definition of the behaviour of the synthesised object.
- These definitions are insufficient. Then a set of behavioural rules must be stated in order to complete the description of the synthesised object. This implies the spec. $\mathrm{n}^{\circ} 10$
Spec. $9 \quad$ For any system, the respect of the specification 7 and of the specification 8 implies the availability of the language for the description of objects belonging to different levels of details. Comment on The concepts of macro-description and micro-description must be introduced there.
Spec. 9
- Example of macro description: the description of cooperating workshops in an enterprise..
- Example of micro description: the description of a hardware device.

Spec. 10 For any given collection of interacting objects, the language must allow the expression of the conditional, or unconditional temporal rules, of their individual actions and of their interactions (physiology of the collection).

Comment on If the given collection constitutes the morphology of an object, then these rules complete the definition of its Spec. 10 behaviour.
Spec. 11 For any given object, the language must allow the definition of a time interval supporting the description of the object.
Comment on This time interval is somewhere upon the real time scale. It is the purpose of the behavioural rules to indicate the moSpec. 11 ment (« lower bound of the interval ») from which the object acts (use of trigger, on/off actuator, ...).
Spec. 12 For any given object, the language must allow the definition of a space location belonging to the description of the object.

Spec. 13 For any given object, the language must allow the topological ${ }^{33}$ description of its set of states within the time interval supporting the definition of the object.
Comment on The set of the states of an object may be continuous or discrete.
Spec. 13
The CSMF standard must specify languages for expressing objects whatever their respective origins. An object may be: simple data, signal (analogue voltage transmitted through a cable, configuration of binary elements transmitted through a BUS, ...), a program, a hardware device, a machine tool, etc.

This item has two aspects:

- The set may be an existing collection.
- The set may be directly defined by collectivising properties. Then it is a template for defining any particular object having these properties.

The analysis of an object implies to examine how the existing lower level objects can act. Three cases are possible:

- This question does not matter immediately.
- The definitions of the lower level objects are sufficient to ensure a complete definition of the behaviour of the synthesised object.
- These definitions are insufficient. Then a set of behavioural rules must be stated in order to complete the description of the synthesised object. This implies the spec. $n^{\circ} 10$ The synthesis of an object is obtained whenever other objects are gathered in order to form its morphology. It is then

33 In this specification the word «topological» deals with the mathematical structure; it does not comply with the definition of IRDS which is too restrictive (bibliogr.: IRDS - part1, p.94).

Spec. 14 For any given object, the language must allow the description of the changes of states and of the association of changes of states.

Comment on The state of an object is an object itself. Then a collection of objects completed with a set of rules of changes for each Spec. 14 pair of objects constitutes a new object.

Spec. 15 The language must allow the description of objects processing other objects. A given object may be, both, an object processing other objects, or an object being processed by an other object.
Comment on Particularly, two important Examples must be emphasised:
Spec. 15

- The transformation of an object having a continuous time support into an other object having a discrete set of time supports (sampling).
- The transformation of an object having a continuous set of states in an other object having a discrete set of states (« quantifying »).
Spec. 16 For any given processing object the language must allow the specification of the exchanges between this object and its environment.


## Production of languages

## 4.1 <br> General

The efficient designer of system description must be attentive of many properties of the described system, but his first concern should be to frame his description elements so that they convey the exact meaning intended. For that purpose, any CSMF must contain a formal description language. This kind of language is usually provided by the used CSMF. We have now to establish a mechanism for validating such a language in respect to the primary formalism.

## 4.2 <br> Rules of meta-production

Many different description languages are available in IT. This paragraph proposes a mechanism for attempting to link any language definition with the primary formalism of the CSMF standard. For that purpose, the notions and the rules of the primary formalism are used as meta-notions and meta-rules. Then some rules, named «rules of meta-production », must be defined especially for any description language L , for linking it with the primary formalism. These meta-rules transform the meta-notion in notions belonging to L. Similarly they transform the meta-rules in rules belonging to L .


Figure 30: Language design

## 4.3

## Conformity criterion

The possibility of defining a set of such rules of meta-production for each given language $L$ is unprovable. In addition, if the primary formalism is an element of the CSMF standard, this possibility is a criterion for proving the compliance of the examined language with the CSMF standard.

An example of description language is given in the Appendix IV, page 63. This language was defined more than ten years before this study. We will attempt now in this appendix to link its definition with the primary formalism defined in this part.

## Part 5

## Conclusion

The introductory quotation from Leibnitz placed before the theoretical elements (3rd part) shows how old this idea is: create a universal language for the statement of any problem and as a medium for solution by reasoning. In his correspondence, Leibnitz often affirmed the existence of this language but did hand down anything to us. With respectful irony, René Thom qualifies this vision today as an «old Leibnitzian dream ». He however suggests, seriously, that the only language having these virtues could quite simply be mathematics.

The study was carried out based upon this idea. In terms of results, it proposes a triptych: a pragmatic method for analysing and modelling information systems based on a catalogue of physical concepts; then, theoretical elements supporting their mathematical modelling; finally, a language for expressing the models built by means of this method and this means.

## Physical concepts

In this approach, the method remains the primary element. We shall rapidly conclude on this subject because the classicism of its elaboration and its results call for few comments. Through a set of physical concepts, the method sets forth a vision of information systems based upon long experience with such systems in scientific and industrial circles. This experience was acquired by personal work and reinforced by the contribution of colleagues working along the same lines. It is however restricted in the end to a field of application with boundaries that are still somewhat fuzzy. Such is the fate of all scientific investigation methods: only long usage will make it possible to solidly assess their value and to determine their limits gradually, as work progresses and years go by.

## Theoretical elements

Our hope now is that the models thus devised will become useful, if not indispensable data for reasoning on the properties of the physical systems investigated. The complexity and the size of the models resulting from the method previously described require a precise and rigorous expression and practically call for the use of powerful information processing facilities.

The chosen means of expression is mathematical modelling. It is a sufficiently powerful tool to meet the requirements of the method and follow the experiences of pragmatic models. Mathematics brings or takes nothing from the assumed fidelity of any model built by means of physical concepts, with regard to the physical being that it represents. The virtues of mathematical modelling lie essentially in the precision and rigor of its definitions, its constructions, and its notations: communication and reasoning, essential parts of any subsequent work, are now based on models void of ambiguity. These models form a family whose general properties have been presented extensively in the part of the study devoted to them.

Information processing facilities are processing systems omnipresent today but have a basic characteristic that must be pointed out here: they are systems with a behaviour that is discrete with respect to time and processing only discrete data sets

What, now, is the possible use of mathematical models? The following proposals concern exclusively any mathematical model of the type proposed in this study and its use as data for computer processing facilities.

For this purpose, let us return for a moment to a classical engineering problem. We all know that the bugbear of a designer or of a system manager is the risk of failures. The identification and nature of possible failures, their causes and their effects are thus of major concern. Simulation techniques provide certain presumptions in this respect during the validation of the systems but never any absolute certainty. Some investigators have consequently turned to methods of formal expertise executed on mathematical models. This is done currently in other areas: a mathematical model, if it is reliable, expresses the qualities of the physical world, its advantages, its drawbacks and, in our case, potential system malfunctioning. Let us briefly examine the nature of the work that can be executed by means of models we know how to build.

Two activities must be distinguished in this examination: that of the mathematician and that of the engineer.

Let us begin with the mathematician's activity and let us consider anyone of the mathematical models concerned.

Let us first eliminate the possibility of any gratuitous disrespect for mathematics in the construction of the model. What is involved would be purely errors of writing. The element of the model having such a fault is most currently void of any meaning; compilers exist ordinarily to detect such errors. There is still the exceptional case in which this disrespect could lead, by extraordinary chance, to a mathematically correct expression which is whimsical in its relations with the modelled physical object. The model would then be like the many other models resulting from imperfect observations of reality, both mathematically correct and physically incorrect. The validation of the models is in fact the function designed to locate and reduce such defects; this classical function forms part of engineering.

After having rid the mathematical model of any mistakes on the part of the writer, we can now assume that there is formal perfection; we have a set of assertions, of complex structure, concerning the physical properties of the investigated system. And, experience warns us that these assertions may contain other contradictions, despite all the preceding precautions. This is even a quite frequent and obviously troublesome case. The formal rectitude of the model now being established, it is the physical rectitude of the physical system that must checked. Guided by these contradictions, unjustly scorned, one generally then discovers latent and insidious risks of inconsistent behaviour, failures and even disasters. It is thus important to reveal and study any contradictions of a mathematical model because they reveal latent ills of the physical system. What procedure shall we use?

We have distinguished two parts in a pragmatic model, conserved in the mathematical model:

- the primary morphology which determines the direct descendants of the modelled physical object;
- the primary physiology which determines the rules of activity of these descendants.

Each descendent is also a physical object capable of being modelled in the same manner, and so on. The reader is referred to the part devoted to physical concepts for details on these definitions.

Primary morphology and physiology are both sets of assertions.
Primary morphology that results from a first analysis of the physical object is a finite sequence of assertions, each meaning intuitively: «x, of the X type, exists ». Each assertion is a true proposition by construction; their sequence represents their conjunction meaning intuitively:
« (a of the A type exists) and (b of the B type exists) and... (z of the Z type exists) ».
The primary morphology is thus globally a true proposition. Consequently, we do not find the contradictions, if they exist, at this modelling level. This assurance is unfortunately of limited value because a primary morphology is only a sort of first inventory determining an initial collection of inert objects whose fine descriptions and animations still remain to be created.

A physical object has different behaviour possibilities that have been modelled using the state concept. To each object corresponds a collection of possible states. The purpose of primary physiology is to define, for each object, the associated primary morphology, the temporal and non-temporal rules of occurrence of these states. Each rule is an action. The legitimacy of these rules does not belong to the world of mathematics; it is an engineering matter. On the other hand, these multiple rules may be gratuitously contradictory in the determination of the state of an object, the different states mutually excluding themselves, and legitimately in this mathematical modelling technique. These contradictions are generally concealed by the overlapping of assertions resulting from the recursiveness of the modelling technique.

Let us review intuitively here, by simplifying to the utmost, the two basic meanings of an action:

$$
\begin{aligned}
& \text { «if « condition } \mathrm{C} \text { » then at } \mathrm{t} \text { « } \operatorname{state}(\mathrm{a})=\mathrm{Ai} \text { » otherwise at } \mathrm{t} \text { ' «state }(\mathrm{a})=\mathrm{Aj} \text { »» ( } \text { (pulse mode) } \\
& \text { «if «condition } \mathrm{C} \text { » then «from } \mathrm{t} 1 \text { to } \mathrm{t} 2 \text { «state(a)=Ai»» otherwise «from t'1 to } \mathrm{t} \text { '2 }
\end{aligned}
$$ « state(a)=Aj » » » (on/off mode).

A model is not a theory in which the different assertions are axioms or theorems deduced from axioms according to the rules of logic. Any relations between assertions express formally a physical reality, which is what distinguishes the model from a demonstration. In particular, the different actions are independent if the physical behaviours represented are independent. On the other hand, they are related by relations translating the relations between the physical components, if necessary. As in any human construction, a system is fallible, and the modelled system may exhibit conceptual errors translated into the model by inconsistencies

The search for contradiction is theoretically simple: for each state « Ai » of an object « a » and for all the actions governing this state directly or indirectly, its characteristic domain is constructed in the space made up of parameters that are involved in its determination. This characteristic domain is the domain « Di » of which «state (a) = $\mathrm{Ai} »$ as a function of these parameters. In order for there to be no contradiction between the assertions determining any « Ai » and « Aj » couple with respect to time, it is necessary and sufficient for the
intersections of the characteristic domains « Di » and « Dj » of these states by the same « plane » to be disjointed whatever t .

Practical research is far more difficult. Determining the intersection of two domains varying with respect to time is possible only if the domains are themselves determined. As processing facilities are digital machines, these domains need only be calculable. And, time and any object «a » are objects that may have complex structures: their respective sets of states may be chosen independently as continuous or discrete depending on modelling requirements. Their calculability at any instant is therefore impossible in the general case. There is in fact a countable, finite and often even highly populated set of calculable cases, but it always remains a set having the power of the continuum of noncalculable cases. The question of knowing whether the countable and finite set of calculable cases is sufficient to provide a satisfactory answer is a question difficult to analyse numerically. It is a sort of discretization of the system into finite elements whose validity, when the solution is found, is an engineering problem ${ }^{34}$.

The severity of this diagnosis is compensated by the fact that certain families of physical systems have models whose properties restrict the problem: for example, so-called discrete sequential systems for which the time and the set of values are discretized. In this case, the assertions are always calculable in principle, and it is then possible to hope to find partial solutions, some of which are in fact found. The accumulation of such solutions and the fact that knowledge on the subject is growing with the years induces the researcher to continue along these lines but should not obscure the theoretical possibility of finding a general solution with numerical processing facilities.

This approach brings us to engineering because it depends on the existence of system families for which its results can be validated.

The examination of the engineer's viewpoint is limited to the examination of two frequent exemplary cases.

The first example is the verification of the compatibility of input and output specifications in the exchange of products between the elements of a system. In a large system, this is typically the first source of failures, which may very reasonably be confined by a systematic examination in the mathematical model of all the morphological conditions of exchanges. This is where it is possible to find the grain of sand forgotten and which will later and quite unexpectedly jam the entire system! By « morphological condition» is meant any specification involved in the definition of a product when it is transmitted and then received. Such an inspection quickly becomes humanly impossible, whereas it can be validly performed by software, not only during the design of the system but also after all the maintenance and upgrading operations which affect its morphology.

The second example is the mutual waiting for two physiologies at a meeting point: an action « a » in one physiology « A » waits for an action « b » in another physiology « B » to be terminated and vice versa. Petri network specialists will recognise this as an example of a partial subgraph which is easy to correct, when one knows where it is hidden. This is a frequent case in complex evolving systems and, as in the preceding case, having a model that evolves with the system makes it possible to locate many points of such blockage.

## Language

Then there is the question of language. Language is designed for the writing of texts easily legible by individuals and machines, while also faithfully expressing the mathematical models.

When the theoretical elements are acknowledged, the definition of the language must comply with the following specification: its vocabulary and its syntax must present faithfully and exhaustively the elements of the theory. This is a stringent rule but practical solutions are possible. In fact, while remaining within the strict framework of this specification, the definition of a language, i.e. of a formal system of notations, has no limits other than those of the imagination.

However, the remark just made with respect to standardisation work has the utility of defining a standard language whose legibility would not necessarily be the main quality. Its purpose would be to serve as an intermediary for any translation of a model expressed in a language based on theoretical elements into an expression in another language different in its appearance, but also complying with the same theoretical elements. This provision favours the creation of languages specific to certain scientific or technical disciplines, the existence of which becomes inevitable owing to the multidisciplinary nature of the systems examined.

Finally, it is noted that the number of problems raised following this study exceeds that of the few questions for which we propose a solution. Having completed this stage, let us put down our instruments. We can take our closing words from either of two great French mathematicians, as the reader chooses.

34 Let us note that Shannon's theorem provides an answer to this question for certain objects: signals.

The first, Bézout, has a style which shows his era. The intense jubilation of having completed something is difficult to hide behind the extreme modesty of pronouncements befitting a decent man of the 18th century.

Nous nous estimerons heureux si considérant le point où nous avons pris les choses, et celui où nous les amenons, on trouve que nous avons acuité une partie du tribut que tout homme doit à la société dans l'état où il se trouve placé. ${ }^{35}$
(We shall regard ourselves as fortunate if, considering the point at which we took things over, and the point to which we shall take them, it is found that we have completed part of the tribute that any individual owes to society.)

The second, which is contemporary because it has to do with Laurent Schwartz, expresses equal satisfaction, judging from the last, clear and limpid line of one of his many valuable works:

$$
\text { Ouf ! }{ }^{36}
$$

(Phew!)

[^5]
## Part 6 <br> ApPendixes

## Action

## SC21 7054

## Action

 IRDS - part1
## Activity

SC21 7054

## Behaviour

SC21 7054

## Behaviour

SC21 9563 (LOTOS)

## Behaviour

SC21 9563
(SDL)
Behaviour
SC21 9563 (Z, p.19)

## Behaviour

SC21 9563
(Estelle, p.25)

## Class

CSMF SOU-07

## Environment

EWOS/ETG 012
ISO TR 9007
Inheritance
(p.961)

Interface
EWOS/ETG 012
ISO 2382-1

## Interface

(p.328)

Interface
SC21 7054

## Interoperability

EWOS/ETG 012
IEEE 729
Interoperability
TSG1
IEEE 729
Location
in space
SC21 7054

## LIST OF DEFINITIONS

Something which happens.
Notes -1 : when used without qualification, action means «action occurrence».
An event whose agent (in this case, an actor) is volitional. Action is a relation among the following: a spatio-temporal location (i.e. a location in space and time); a volitional agent; a phenomenon; an instrument; and an act. One or more elementary actions that, as a unit, change a collection of sentences into another collection of sentences in the information base or conceptual schema and/or make known a collection of sentences present in the information base or conceptual schema. (Page 90).
A single-headed directed acyclic graph of actions where occurrence of each action in the graph is made possible by the occurrence of all immediately preceding actions.
A specification of a set of activities.
... is defined by the LOTOS behaviour expression associated with the process definition that constitutes the object template. ..

The behaviour of a process/service is the set of all transitions of that process/service. ..

The behaviour of an object in a given state is the set of all possible activities that may occur from that state. ....

The behaviour of an object is determined by the set of all transitions of that object. ...

A class is an abstraction of objects having common properties. Such an object is called an instance of the class. When an object is an instance of a class, we say the object belongs to the class
(of information system). That part of the real world containing the users which exchange messages with the information system.
Inheritance is a mechanism for sharing code and behaviour. It allows to reuse the behaviour of a class in the definition of new classes. Subclasses of a class inherit the operation of their parent class and may add new operation and new instance variables.
A shared boundary between two functional units, defined by functional characteristics, common physical interconnection characteristics, signal characterisrics, or other characteristics, as appropriate.
... To work in a system, every product must comply with precise rules governing its intended relationships with other products, with user software, and even wuth user interactions. Such relationships are called interfaces. ...
An abstraction of the behaviour of an object obtained by considering only a specified subset of the observable actions of that object.

The ability of two ore more systems to exchange information and to mutually use the information that has been exchanged. (IEEE 729).
The ability of two ore more systems to exchange information and to mutually use the information that has been exchanged. (IEEE 729).
An interval of arbitrary size in space at which an atomic action can occur.

Location in time
SC21 7054

## Location

SC21 7054
Object

Object
Object
SC21 7054
Object
CSMF SOU-07
Object
IRDS-part 1
Object
ISO/IEC JTC1
TC22 N1712
Object
ISO/IEC JTC1
TC22 N1712
Portability
TSG-1

## Portability

EWOS/ETG 012
TSG-1
Portability
TSG-1
Portability
EWOS/ETG 012
TSG-1
Portability
EWOS/ETG 012
ISO-2382-1

## Process

IRDS-part 1
ISO TR9007
State
SC21 7054
(p.328)

System
EWOS/ETG 012
OED
System
BSI DD 210
OED
System
IRDS-part 1
Template
SC21 7054
Type
SC21 7054
Universe of Dis-

## course

IRDS-1

System ..., a system appears as a coherent collection of products, both hardware and software. .
An interval of arbitrary size in time at which an atomic action can occur.
An interval of arbitrary size in time and space at which an atomic action can occur.
Objects in programming languages are collections of operations that share a state.
The collection of methods of an object determines its interface and its behaviour. Object denotes the whole representation of an application data model. ...

A model of an entity. An object is characterized by its behaviour and, dually, by its state (...)
An object is a representation of a real world object. An Object has an identifier that identifies itself in an application data model occurrence. ...
Something toward which a cognitive act (i.e. a though) or an action is directed. A material body in the mid-world.
(External). Hardware and/or software unit that has a behaviour and interaction modes, with Ada programs, defined by this standard
(Logical). Conceptually, a software object that is created and associated with an external object and that has an acccessor. [Software object is not defined in this document].
(Software). The ease with which software can be transferred from one application platform to another.
(Software). The ease with which software can be transfered from one information processing system to another.
(Application). The ease with which an application can be transferred from one application platform to another.
(Application). The ease with which an application can be transferred from one application platform to another.
(Program). The capability of a program to be executed on various types of data processing systems without converting it to a different language and with little or no modification.
A collection of activities performed in a set order on a prescribed set of constructs under the constraint of rules.
At a given instant in time, the condition of an object that determines the set of all sequences of actions in which the object can take part.

A set of connected things, parts, elements working together in a regular relation.
A set of connected things, parts, or elements working together to achieve a common objective.
Ordered set of ideas, concepts, principles.
(business). A system which performs one or more of the tasks of one ore more business funcions by transforming a set of inputs, using a set of rules and procedures, to produce a set of outputs.
A methodical or logical plan or arrangement governed by a set of principles, or business rules and procedures.
The specification of the common features of a collection of objects.
(of an object). A predicate. An object is of the type, or satisfies the type, if the predicate holds for the object.
Those entities and happenings of interest that have been, are or ever might be and about which there exists a collection of represented information having a common understanding.

## II. 1

Category

This part concerns the use of the theory of categories. This theory is certainly one of the most abstract approaches of modern algebra. The understanding of its axiomatic foundations is difficult but essential for its strict overall use. In particular, the axioms distinguish two concepts: classes and sets, in order to avoid certain well-known paradoxes of set theory.

Nevertheless, category theory introduces the restrictive notion of "small category", which is simpler than the general notion of "category" and better suited to our needs. The following elements provide a review of the theory which is sufficient for this project.

The modelling of a system generally requires the engineer to state his physical hypotheses concerning the system; these hypotheses are assertions concerning the natural properties of the system, making it possible to choose the abstract elements representing the concrete elements of the investigation domain. We shall now examine how mathematical elements can be used for this purpose.

## II.1.1 Definition

def. 17: Consider an entity $\mathbf{C}^{37}$; this entity is a "small category" if we can assign the following properties to it ${ }^{38}$.
a) Initial definitions

Considering,

def.20: a function •: mor (X,Y)× $\operatorname{mor}(\mathrm{Y}, \mathrm{Z}) \rightarrow \boldsymbol{\operatorname { m o r }}(\mathrm{X}, \mathrm{Z})$, associated with any triplet $(\mathrm{X}, \mathrm{Y}, \mathrm{Z})$ such as $\operatorname{mor}(\mathrm{X}, \mathrm{Y}) \neq \varnothing$ and $\operatorname{mor}(\mathrm{Y}, \mathrm{Z}) \neq \varnothing$ and $\operatorname{mor}(\mathrm{X}, \mathrm{Z}) \neq \varnothing$;
this function determines a composition:
if $\mathrm{f} \in \operatorname{mor}(\mathrm{X}, \mathrm{Y})$ and $\mathrm{g} \in \operatorname{mor}(\mathrm{Y}, \mathrm{Z})$, then there is exactly one $\mathrm{k} \in \operatorname{mor}(\mathrm{Y}, \mathrm{Z})$ and $\mathrm{k}=\mathrm{g} \bullet \mathrm{f}$.

[^6]
## b) Axioms

axiom 1: the composition $\bullet$ is associative;
if $\quad \mathrm{f} \in \operatorname{mor}(\mathrm{X}, \mathrm{Y})$ and, $\mathrm{g} \in \operatorname{mor}(\mathrm{Y}, \mathrm{Z})$ and, $\mathrm{h} \in \operatorname{mor}(\mathrm{Z}, \mathrm{W})$,
then $\quad(h \bullet g) \bullet f=h \bullet(g \bullet f)$;
axiom 2: for any element Y , there is a $\operatorname{mor}(\mathrm{Y}, \mathrm{Y})$ having at least one element $\mathrm{u}_{\mathrm{Y}}$; this element is such that:
for any X and any $\mathrm{f} \in \operatorname{mor}(\mathrm{X}, \mathrm{Y}) \neq \varnothing$,
$\mathrm{u}_{\mathrm{Y}} \bullet \mathrm{f}=\mathrm{f}$,
and,
for any Z and any $\mathrm{g} \in \operatorname{mor}(\mathrm{Y}, \mathrm{Z}) \neq \varnothing$,

$$
\mathrm{g} \bullet \mathrm{u}_{\mathrm{Y}}=\mathrm{g}
$$

c) Comment

In the definition (def 19), the case of the existence of an empty set of morphisms must be discussed.

If $\operatorname{mor}(\mathrm{X}, \mathrm{Y})=\varnothing$ (resp. $\operatorname{mor}(\mathrm{Y}, \mathrm{Z})=\varnothing$ ), then $\mathrm{f}($ resp. g$)$ is nonexistent; its composition with any other morphism is consequently nonexistent. This circumstance is compatible with the axioms. Let us now suppose that this $\operatorname{mor}(\mathrm{X}, \mathrm{Y}) \neq \varnothing$ and $\operatorname{mor}(\mathrm{Y}, \mathrm{Z}) \neq \varnothing$ but that $\operatorname{mor}(\mathrm{X}, \mathrm{Z})=\varnothing$. This circumstance, possible in a model, leads to a contradiction with the existence of the composition law. The associativity axiom is contradicted, and consequently the model is no longer a category.

## II.1.2 Opposite category

def. 21: Let $\mathbf{C}$ be a category. The opposite category $\mathbf{C}^{\mathbf{0 p}}$ is such that:

- obj( $\left.\mathbf{C}^{\mathbf{0 p}}\right)=\mathbf{o b j}(\mathbf{C})$;
- there is a bijection between $\operatorname{mor}(\mathrm{X}, \mathrm{Y})$ and $\operatorname{mor}^{\mathbf{0 p}}(\mathrm{Y}, \mathrm{X})$.


## sentation of $\mathbf{C}$.

Example 38: A series of data having a "skip ${ }^{+}()$function is a category if skip ${ }^{+}(0)$ and skip ${ }^{+}(n+m)=$ skip $^{+}(m) \bullet$ skip $^{+}$(n) are defined. The opposite category is obtained with the function skip ${ }^{-}()$.

## II.1.3 Product of categories

Let $\mathbf{C}$ and $\mathbf{C}^{\prime}$ be two categories.
def. 22: The product $\mathbf{C}^{\prime \prime}=\mathbf{C} \times \mathbf{C}^{\prime}$ is such that:

- obj( $\left.C^{\prime \prime}\right)=\mathbf{o b j}(C) \times o b j\left(C^{\prime}\right)$,
- if $f \in \operatorname{mor}(X, Y)$ in $\mathbf{C}$, and $f^{\prime} \in \operatorname{mor}\left(X^{\prime}, Y^{\prime}\right)$ in $\mathbf{C}^{\prime}$, then $\mathrm{f}^{\prime \prime}=\left(\mathrm{f}, \mathrm{f}^{\prime}\right) \in \boldsymbol{\operatorname { m o r }}\left(\left(\mathrm{X}, \mathrm{X}^{\prime}\right),\left(\mathrm{Y}, \mathrm{Y}^{\prime}\right)\right)$,



## II.1.4

## Ordered set

The ordered set is an example of an entity assimilable with a category; this example is essential in our methodology.

Let us consider a set $S=\{X, Y, \ldots\}$. $S$ is ordered if there is a binary relation " $\leq$ " which is transitive, associative and reflexive, and a set $\mathrm{O} \subset \mathrm{S} \times \mathrm{S}$ such that:

$$
\mathrm{O}=\{(\mathrm{X}, \mathrm{Y}) / \mathrm{X} \leq \mathrm{Y}\}
$$

(i) If $\mathbf{C}$ is the entity, we declare that:

- $\mathrm{S}=\mathbf{o b j}(\mathbf{C})$;
- for any pair $(\mathrm{X}, \mathrm{Y}) \in \mathrm{S} \times \mathrm{S}$ there is $\operatorname{mor}(\mathrm{X}, \mathrm{Y})$, which is either empty or equal to the singleton $\{\leq\}$;
- The transitivity of $\leq$ leads to the existence of: $\bullet: \operatorname{mor}(\mathrm{X}, \mathrm{Y}) \times \operatorname{mor}(\mathrm{Y}, \mathrm{Z}) \rightarrow \boldsymbol{\operatorname { m o r }}(\mathrm{X}, \mathrm{Z})$.
(ii) Let us note that:
- The transitivity of $\leq$ implies the associativity of $\bullet$.
- Reflexivity implies the existence of the singleton $\left\{\mathrm{u}_{\mathrm{X}}\right\}=\boldsymbol{\operatorname { m o r }}(\mathrm{X}, \mathrm{X})$ for any $\mathrm{X} \in \mathrm{S}$.


## II. 2

Fonctor

## II.2.1 <br> Definition

def. 23: Let $\mathbf{F}$ be an entity; this entity is a fonctor if we can assign the following properties to it.
a) Notations

Let $\mathbf{C}$ and $\mathbf{C}^{\prime}$ be two categories and F an: $\mathbf{o b j}(\mathbf{C}) \rightarrow \mathbf{o b j}\left(\mathbf{C}^{\mathbf{\prime}}\right)$.
Let $X$ and $Y$ be any two categories of $\mathbf{C}$ and their respective images $X^{\prime}$ and $Y^{\prime}$ in the objects of C'.

## b) Axioms

axiom 3: for any couple ( $\mathrm{X}, \mathrm{Y}$ ) F determines an application $\mathrm{F}: \operatorname{mor}(\mathrm{X}, \mathrm{Y}) \rightarrow \boldsymbol{\operatorname { m o r }}\left(\mathrm{X}, \mathrm{Y}^{\prime}\right)$ with the condition:
if $Y=f . X$, then $Y^{\prime}=f^{\prime} . X^{\prime}$ with $f^{\prime}=F(m)$.
axiom 4: F conserves the morphism composition law:
if $k=g \bullet f$, then $k^{\prime}=g^{\prime} \bullet f$,
The following equalities result from this axiom:
Z' = k'. $\mathrm{X}^{\prime}=\left(\mathrm{g} ’ \bullet f\right.$ '). $\mathrm{X}^{\prime}=\mathrm{g}^{\prime} . \mathrm{Y}^{\prime}$
axiom 5: F conserves the associativity of the composition law:
if $k{ }^{\prime}=h^{\prime} \bullet g^{\prime} \bullet f^{\prime}$, then $k^{\prime}=\left(h^{\prime} \bullet g^{\prime}\right) \bullet f^{\prime}=h \prime \bullet\left(g^{\prime} \bullet f \prime\right)$
The following equalities result from this axiom:
$W^{\prime}=l^{\prime} . X^{\prime}=\left(h \prime \bullet g \prime \bullet f^{\prime}\right) . X^{\prime}=\left(h^{\prime} \bullet g^{\prime}\right) . Y^{\prime}=h^{\prime} . Z^{\prime}$
II.2.2

## Natural transformation of a fonctor

def. 24: Let $\mathbf{F}$ and $\mathbf{G}$ be two functions such that: $\mathbf{F}, \mathbf{G}: \mathbf{C} \rightarrow \mathbf{C}$. $\tau$ is a natural transformation if it associates with any $\mathrm{X} \in \mathbf{o b j}(\mathbf{C})$ a morphism $\tau_{\mathrm{x}}: \mathbf{F}(\mathrm{X}) \rightarrow \mathbf{G}(\mathrm{X})$ such that for any $\mathrm{f} \in \boldsymbol{\operatorname { m o r }}(\mathrm{X}, \mathrm{Y})$ the following diagram commutes:


## II.2.3 Universal element

def. 25: Consider a fonctor $\mathbf{F}$ : $\mathbf{I} \rightarrow \mathbf{O}$. The objects of $\mathbf{I}$ and $\mathbf{O}$ are sets. A universal element $\mathbf{F}$ is a pair ( $u, R$ ) composed of an object $R \in \mathbf{o b j}(\mathbf{I})$ and an element $u \in \mathbf{F}(R)$ having the following property: for any object $X \in \mathbf{o b j}(\mathbf{I})$ and any $s \in \mathbf{F}(X)$, there is a unique morphism $f: R \rightarrow X$ such that $\mathbf{F}(\mathrm{f}) \mathrm{u}=\mathrm{s}$.

Example 39: Consider two countable series having the number of records $n$ : $\mathrm{D}=\left\{\mathrm{d}_{\mathrm{i}}\right\}, \mathrm{N}=\left\{\mathrm{n}_{\mathrm{i}}\right\}$. Now consider a category C such that: $\mathbf{o b j}(\mathbf{C})=\{\mathrm{N}, \mathrm{D}\}$,
$\operatorname{mor}(\mathrm{N}, \mathrm{N})=\operatorname{mor}(\mathrm{N}, \mathrm{D})=\boldsymbol{\operatorname { m o r }}(\mathrm{D}, \mathrm{N})=\boldsymbol{\operatorname { m o r }}(\mathrm{D}, \mathrm{D})=\left\{\Omega_{\mathrm{i}}\right\}$.
Any $\Omega_{\mathrm{i}}$ is a circular permutation matrix $\left[\varpi_{\alpha \beta}\right]$ in which $\varpi_{\alpha \beta}=1$ if $\alpha=\beta-\mathrm{i}+1\left(\bmod \alpha_{\max }\right)$, otherwise 0 . These permutation matrices are composable and this composition is associative; the result is a cyclic permutation matrix; this composition is associative. For any $\mathrm{X}(\mathrm{X}=\mathrm{D}$ or N$)$, we say that $\Omega_{\mathrm{i}}$ : $\mathrm{N} \rightarrow \mathrm{X}$ selects exhaustively the record i in X . This assertion is obviously analogous to $\Omega_{\mathrm{i}}$ : $\mathrm{n}_{1} \rightarrow \mathrm{x}_{\mathrm{i}}\left(=\mathrm{n}_{\mathrm{i}}\right.$ or $\left.\mathrm{d}_{\mathrm{i}}\right)$. We now state that N is a given permutation (not restricted to a cyclic permutation) of $\operatorname{mor}(\mathrm{N}, \mathrm{D}): \mathrm{N}$ appears as an index file for the file D . We give a transformation represented by the fonctor $\mathbf{F}$. We choose F such that: $\mathbf{F}\left\{\Omega_{\mathrm{i}}\right\}=\left\{\Omega_{\mathrm{i}}\right\}$,
any $\mathbf{F}\left(\Omega_{\mathrm{i}}\right): \mathbf{F}(\mathrm{N}) \rightarrow \mathbf{F}(\mathrm{X})$ points to the record i in $\mathbf{F}(\mathrm{X})$.

## II.2.4

## Representation

def. 26: Consider a fonctor $\mathbf{F}$ : $\mathbf{I} \rightarrow \mathbf{O}$. The objects of $\mathbf{I}$ and $\mathbf{O}$ are sets. A representation for $\mathbf{F}$ is a pair $(\mathrm{R}, \varphi)$ composed of an object $\mathrm{R} \in \mathbf{o b j}(\mathbf{I})$ and of a family of bjiections $\varphi \mathrm{X}: \operatorname{mor} \mathrm{X}(\mathrm{R} \rightarrow \mathrm{X}) \equiv$ F(X).
The latter condition implies that, if $v_{1}=\mathbf{F}\left(n_{1}\right)$, then for any $i, x_{i}=\mathbf{F}\left(\Omega_{i}\right) v_{1}$. Consequently, $\left(v_{1}, N\right)$ is universal for $\mathbf{F}$.

## dem.1: Temporal translation of a process

The processes, elements of $\gamma^{\prime}{ }_{\tau+t}$ represent the same phenomena as the processes which are elements of $\gamma_{\tau}{ }_{\tau}$ to within a translation $t$ with respect to time. Conversely, any value of $t$ determines in $\Gamma^{\prime}$ an operator transforming each $\gamma^{\prime}{ }_{\tau}$ into its translation $\gamma^{\prime}{ }_{\tau+\mathrm{t}}$. Consequently, there is $\Gamma_{\mathrm{t}}: \gamma_{\tau}{ }_{\tau} \rightarrow \gamma^{\prime}{ }_{\tau+\mathrm{t}}$.

In particular, for $\tau=0, \Theta_{\mathrm{t}}: \gamma^{\prime}{ }_{0} \rightarrow \gamma^{\prime}{ }_{\mathrm{t}}$.

## dem.2: Sampling and quantization

- $\tau \in\left[\tau_{1}, \tau_{2}\right] \cap\left[\mathrm{t}_{1}, \mathrm{t}_{2}\right]$

Hypothesis 12 leads to:
$\delta[\tau] \otimes g^{\prime \prime}=\delta[\tau] \otimes\left(\delta[\tau] \bullet g^{\prime}\right) ;$
Hypothesis 13 entails directly: $\delta[\tau] \otimes\left(\delta[\tau] \bullet g^{\prime \prime}\right)=\delta[\tau] \otimes(\delta[\tau] \bullet g) ;$
Hypothesis 12 again leads to: $\quad \delta[\tau] \otimes(\delta[\tau] \bullet g)=\delta[\tau] \otimes \mathrm{g}$; consquently, the first relation is demonstrated.

- $\tau \notin\left[\tau_{1}, \square \tau_{2}\right] \cap\left[\mathrm{t}_{1}, \mathrm{t}_{2}\right]$

This condition leads to $\delta[\tau] \bullet g^{\prime \prime}=\delta[\tau] \bullet g=\delta[\tau] \bullet g 0$, which enables us to substitute $g_{0}$ for $g$ in all the above relations; the second relation is thus also demonstrated.

## dem. 3: Topology of $\Gamma$,

By construction, there is a bjiection between T and $\Gamma^{\prime} ; \Gamma^{\prime}$ consequently has the same topology as T and consecutively the power of the continuum. Let us note this topology as $\mathrm{T}_{\mathrm{T}}\left(\Gamma^{\prime}\right)$.

## dem. 4: Topology of substrate space

Let us consider a particular process $g$ of domain $\left[t_{1}, t_{2}\right]$. For any interval $\left[t_{1}, \tau_{2}\right]$ included in the preceding (or equal), there is $g^{\prime \prime}=\Delta[t 1, \tau 2] \bullet g$. By construction, the set $\left\{g^{\prime \prime}\right\}$ has the same power as $\left[t_{1}, t_{2}\right]$, i.e. the power of the continuum; also by construction, the elements of $\{\mathrm{g}$ ' $\}$ \}are moreover different two by two. But, the definition of the equivalence relation $\rho$ indicates that any element of $\left\{g^{\prime \prime}\right\}$ has an equivalent $g$ ' ' in $\gamma^{\prime}{ }_{0}$; still by construction, these elements are different two by two; their set $\left\{g^{\prime \prime}\right\}$ has at least the power of $\left\{g^{\prime \prime}\right\}$. $\left\{g^{\prime \prime}\right\}$ is a subset of $\gamma^{\prime}{ }_{0} ; \gamma^{\prime}{ }_{0}$ thus has at least the power of the continuum.

## dem. 5: Portability

Let us note first of all that any symbol may be chosen in the definition 20 (Page 22); in particular, we can permute the symbols $\mathbf{F}$ and $\mathbf{G}$ without altering this definition.

- The condition is sufficient:

If all the $\tau_{\mathrm{X}}$ are invertible, the inversion of the morphisms leads to the plotting of diagrams identical to that of the definition, allowing for the symbols chosen. The preceding remark means that we find the definition of the natural transformation.

- The condition is necessary:

Let us suppose that a single morphism is not invertible, for example $\tau_{\mathrm{X}} \mathbf{F}(\mathbf{X}) \rightarrow \mathbf{G}(\mathbf{X})$. In this case, the construction of the diagrams shows that the transformation is in conformity with the definition for $\tau_{\mathrm{X}}$ and $\tau_{\mathrm{Y}}$ and false for $\tau_{\mathrm{X}}$ and $\tau_{\mathrm{Y}}{ }^{-1}$. The supposition leads to the impossibility of applying definition 22 . The condition is thus necessary.

## General

There is an example of production of a description language named «Dad». This example was first published in its preliminary version in $19833^{39}$. A revised version was proposed in an appendix of a report on the use of Ada for system description. This report was submitted to the Commission of the European Communittee in 1984 and published by the Cambridge University Press in 1985, under the title « Ada for specification: possibilities and limitations » ${ }^{40}$.

## IV. 2

## Basic rules of production

The rules of production are those used in the definition of Ada in its first standardized version ${ }^{41}$. The production rules for which we propose adaptations are given only after that introduction, labelled with their initial reference. Within these new rules, the notions or the parts of notions chosen as the roots of the extensions are indicated by capital letters. The meta-rules and rules of meta-production associated with these roots are given in the two following sections.

## IV. 3 Rules of production

Bold-face references in the first column are paragraph numbers in the referenced Ada man-

| 2.8. | REFERENCE | : : $=$ | REFERENCE'SYMBOL identifier [(argument-association \{, argument-association\}) |
| :---: | :---: | :---: | :---: |
| 3.1. | basicdeclaration | : : $=$ | object-declaration |
|  |  |  | / FLUX-declaration |
|  |  |  | / DEVICE-declaration |
|  |  |  | / PROCESS-declaration |
|  |  |  | / exception-declaration |
|  |  |  | / renaming-declaration |
|  |  |  | / number-declaration |
|  |  |  | / SUBFLUX-declaration |
|  |  |  | / SYSTEM-declaration |
|  |  |  | / generic-declaration |
|  |  |  |  |
|  |  |  | / defered-constant-declaration |
| 3.2. | objectdeclaration | : : $=$ | identifier-list : [constant] SUBFLUX-indication [:= expression] |
|  |  |  | / identifier-list: [constant] constrained-array-definition [:= expression] |
| 3.3.1. | FLUX-declaration : := |  | full-FLUX-declaration |
|  |  |  | / incomplete-FLUX-declaration |
|  |  |  | / private-type-declaration |
|  | full-FLUX- | : : = | FLUX'SYMBOL identifier [discriminant-part] |
|  | declaration |  | is FLUX-definition ; |
|  | FLUX-definition | : : $=$ | enumeration-definition |
|  |  |  | / real-type-definition |
|  |  |  | / record-type-definition |
|  |  |  | / derived-type-definition |
|  |  |  | / integer-type-definition |
|  |  |  | / array-FLUX-definition |
|  |  |  | / access-type-definition |
| 3.3.2. | SUBFLUX- | : : $=$ | SUBFLUX'SYMBOL identifier is SUBFLUX-indication ; |

39 Savoysky, 1983. Eléments théoriques pour la description de systèmes automatiques. Thèse de Doctorat d'Etat, Univ. Pierre et Marie Curie, Paris-6, 1983.
40 Goldsack and al, 1985: xvii.
41 Alsys, 1983: ANSI/MIL-STD 1815 A.

Appendixes



## Metarules

```
ACTION : STATEMENT,GROUP_ITEM,LAST_GROUP_ITEM
BASIC ORDER
[[PREPOSITION_PART] VERB_PART]
GROUP
ORDERED_ACTION
ORDERED_ACTION
STATEMENT,[{GROUP_ITEM} LAST_GROUP_ITEM]
BASIC_ORDER,BASIC_ORDER (ORDERED_ACTION),BASIC_ORDER name
{PREPOSITION'SYMBOL [expression]}
STATEMENT_SEPARATOR,GROUP_ITEM_SEPARATOR,
LAST_GROUP_ITEM_SEPARATOR
COMMANND'SYMBOL,TIME_OPERATOR'SYMBOL,OTHER_OPERATOR'SYMBOL
```


## Rules of meta-production

| COMMAND'SYMBOL | abort | initiate, run |
| :---: | :---: | :---: |
| DEVICE | subprogram | device, subsystem |
| DEVICE'SYMBOL | procedure, function | device, subsystem, component, binder, adapter |
| EXCHANGE | accept | receive, send, connect |
| EXCHANGE'SYMBOL | accept | receive, send, connect |
| EXCHANGER | entry | receiver,sender,connector |
| EXCHANGER'SYMBOL | entry | receiver, sender, connector |
| FLUX | type | category, flux |
| FLUX'SYMBOL | type | category, flux |
| GROUP_ITEM |  | action |
| GROUP_ITEM_SEPARATOR |  | , |
| ITERATIVE | loop | repeat |
| ITERATIVE'SYMBOL | loop | repeat |
| LAST_ACTION | statement | action |
| LAST_GROUP_ITEM |  | action |
| LAST_GROUP_ITEM_SEPARATO |  | ; |
| R |  |  |
| MORPHOLOGICAL | declarative | morphological |
| OPERATOR'SYMBOL |  | hold, sample, translate |
| PREPOSITION'SYMBOL |  | since, to, at, until |
| PROCESS | task | fonctor, process |
| PROCESS'SYMBOL | task | fonctor, process |
| REERENCE'SYMBOL | pragma | model |
| REFERENCE | pragma | model |
| STATEMENT | statement |  |
| STATEMENT_SEPARATOR | ; |  |
| SUBFLUX | subtype | subcategory, subflux |
| SUBFLUX'SYMBOL | subtype | subcategory, subflux |
| SYSTEM | package | system |
| SYSTEM'SYMBOL | package | system |
| TIME_OPERATOR'SYMBOL |  | hold, sample, translate |

## IV. 6 Tutorial for expressing a behaviour

The notions developed in this section were presented at different states of progressions in some IFAC/IFIP workshops on real-time systems and published by the IFAC. A more complete and achieved presentation was first given in French, in the thesis of the author, and was after inserted in a report of a study group, submitted to the Commission of the European Communities. Most of the comments thereafter are extracted from this report.

## IV.6.1 General conventions

First, a basic distinction between the notion of «behaviour» and that of «sequence of statements » must be done. Assume the following sequence written in Ada:

Example 40: Measure:= Sample(Voltage);
accept Output (Measure: out Pulse);
This sequence is assumed to belong to the description of a concrete-process command expressed, say in the form of a control program; it defines two of the statements of this program, to be executed in the order in which they are written. Now, assume a quite similar expression in the description language Dad:

```
Example 41: Measure:= sample Voltage ,
    send Output (Measure: out Pulse);
```

Under the convention we propose, this text no longer belongs to the description of the con-crete-process command but to the description of a group of possible actions of the concrete process itself. We describe in this example how the element Measure is emitted as it is produced. For this purpose we write, not two statements that are runable sequentially, but two relationships, simultaneously true, between the interactive elements of a system that exists continuously in time. We now present the essential conventions chosen for Dad, with the understanding that this treatment does not claim to be exhaustive.
(I) Any behaviour is a sequence of one or more different groups of actions ordered in time; their expressions are separated by semi-colons.
(II) Every group is a set of one or more concurrent actions; the expression of actions within a group are separated by commas.
Example 42 begin
-- this is the first group of parallel actions:
Measure(1):= sample Voltage(1), Measure(2):= sample Voltage(2), initiate Measures_Edition; -- This is the second group following in time the previous one: Measure(1):= sample Voltage(1), Measure(2):= sample Voltage(2), end;

We will use the qualifiers «descendent» and «parent» in order to situate reciprocally the groups and actions which may be recursively embedded.

## IV.6.2

## Real-Time

Any element of a system, and in consequence the system as a whole, is intended to function: the « action» is the expression specifying its possible ways of functioning. Any element implies the existence of a first form of modelling, based on a substrate space (see definition: page 29) of which each element defines, up to a translation in time, a functioning of the represented element of the system. Specifying all the possible functionings of this element therefore consists of localising its first model in time by a command and possibly, of specifying any modification made by an operator to this model at the amount of localisation. An action belongs to a group and any group belongs to the behaviour of an element, itself activated by at least one action, except for the system as a whole.

## A) Command, significant instant

The localisation in time of a group of actions in a behaviour or of an action in a group is specifying by a « command », sometime implicit. Once again, we use the first form of modelling (category) to specify its use.

First, let us consider a single action.

Example 43: at T_Start run X;
X is a part of a system; X is represented by a category $\mathbf{C}$. Each element of the space, subjacent to the definition of the category has a time domain; their reunion is the domain of X . Hence, any definition of such a category $\mathbf{C}$ implies the existence of a time axis $\theta$ intrinsic to that item and supporting the domain of $X$; let $0(\theta)$ the origin of this axis. A command localises the category $\mathbf{C}$ in time; the result is a commanded action; then the command localises $0(\theta)$ on the time axis T associated with the parent action of the commanded action; the instant $\mathrm{t}_{\mathrm{c}}$ of localisation on this axis T is known as the instant of command. Progressing in this way, the domain of each activated part X of the system is situated on the time axis associated with the whole system which represents the absolute time. The lower bound of this domain is the instant of activation; the upper bound is the instant of inhibition. In the example 43, T_Start is the instant of command and the instant of activation; the instant of inhibition is generally the instant of the normal end of $X$. The instant of activation may be different of the instant of command like in the example 44 thereafter.

Let us now consider a group of actions activated with the same command. The instant of activation of the group is the lowest bound of the different associated domains and the instant of inhibition is the latest of their upper bounds.

The instant of activation and the instant of inhibition of a group of actions are the significant instants of this group.

## B) Operator

The localisation in time is often accompanied by modifications of the model representing modifications of temporal properties of the part of the system; these modifications are specified by an operator. They may concerns the specification of the instants of activation and inhibition.

Example 44: at T_Start run (since T_1 until T_2 sample X);

## C) Critical and free time-part of an action

This dependency among groups and actions, previously qualified as «descendent» or «parent», implies relationships between significant instants. The distinction between a parent entity and its descendent entities, together with the definition of the time relations among them, depends on the conventional meaning we attach to the terms used in their expressions: we examine these conventions in the paragraph that follows. We firs posit the following conventions:
(I) The intrinsic time axis of the system as a whole is chosen as the absolute axis; its origin is the absolute origin. This origin represents the instant of command for activating the entire system. We state that this command at the highest level is always implicit
(II) Let be a sequence of groups of actions embedded in a parent action. For the first group, the instant of command and of activation is the instant of activation of the parent action. For any other group the instant of command and of activation is the end of inhibition of the critical part of the preceding group.
(III) For all action produced by a single parent group, the instant of command, in the absence of any condition or of any specification on this instant, is the instant of activation of the group. The instants of activation and of inhibition depend of the conventional meaning we attach to the terms used in the action expression.
(IV) Any action has a portion that is critical for its parent group and which begins at the instant of command; its normal duration, in the absence of inhibition of the parent action by an abort, is the by the meaning of the terms used in its expression; the instant that is the latest of all the ends of the critical portions of actions having the same parent group sets the earliest instant of normal inhibition of that parent group. Note that the duration of this critical portion may be null.
(V) Any critical portion may be followed by a free portion. The instant at which the free portion begins is the instant at which the critical portion preceding it ends; the instant at which it normally ends, in the absence of inhibition of the parent action by abort, is set by the meaning of the terms used in its expression.
(VI) The inhibition of an action by abort implies the recursive inhibition of the descendent or derived groups and actions that would not yet normally inhibited. Any free portion is without effect upon the instant of normal inhibition of the parent group.

## IV.6.3 Creation and modification of an active element

The purpose is to specify the production of new elements in the system and to active these new elements .

The «assignment_ACTION » rule is used for this purpose. The first member designates the element created or modified. The second member specifies a combination of elements that are already active and of which the significant instants are already determined.

## IV.6.4 Simple action

The purpose of these expressions is to activate the elements of system by ordering them, and, possibly, to transform them.

These expressions are all derived from the rules of metaproduction proposed for the «ORDERED_ACTION » The expressions obtained by the use of this rule are extensions of Ada, except for the expression « abort-statement ». For each action, the significant instants and the bound between the critical and free portions result from the meaning attached by convention to the term used.

We distinguish basically two kinds of simple actions: the triggered action and the on/off action.

## A) Triggered action

The expression of a triggered action uses basically the command «initiate » preceding the name of the element which is activated.

Example 45: initiate Device;
The instant of activation is the instant of command. The instant of inhibition is the instant of normal end of functioning of the activated element or the instant of an abort concerning it. The triggered action is entirely free for its parent and for the following action in a sequence.

The operator initiate may be implicit. As well the expression «Device » is equivalent to the expression «initiate Device».

```
Example 46: initiate Device ,
initiate Other_Device;
Example 47: initiate Device;
initiate Other_Device;
```

Let us note that in the examples 46 and 47 above, the instant of command of the second action is equal to the instant of command of the first action.

## B) On/off action

The expression of a on/off action uses basically the command « $\mathbf{r u n}$ » preceding the name of the element which is activated.

```
Example 48: run Device;
Example 49: run Device;
    run Other_Device;
Example 50: run Device,
run Other_Device;
```

The instant of activation is the instant of command. The instant of inhibition is the instant of normal end of functioning of the activated element or the instant of an abort concerning it. The on/off action is entirely critical for its parent and for the following action in a sequence.

In the example 49 the instant of command and of activation of the second action is the end of the preceding. In the example 50 the two actions are parallel and their respective instants of command and of activation are equal.

## IV.6.5 Synchronisation of behaviours

## A) Exchange action

The exchanges actions are used for that purpose. An exchange action may involves a derivative action. The instant of command (receive/send) is the instant of activation of the parent group. The instant of activation is the instant of command. The instant of inhibition is at the earlier of:
(I) the instant of inhibition of the parent group by abort,
(II) the latest end of the exchange of a maximum form for each element of the formal list.

The exchanger is required to recognise a form in time for each of the element exchanged defined in the formal lit. The formal list specifies, for each element exchanged, a category model; this model determines various possible forms for each element exchanged which are the elements of the substrate space. Each element of the substrate space has an associated time interval which is its domain. The element having a domain of which the upper bound is less the upper bound of all other domains is called a minimum form; correspondingly one can define a maximum form. For each exchanger action, the instant of activation of the derivative action is considered ion addition to the significant instant: this instant ends the minimum duration
needed to exchange a minimum form for each element of the formal list. In general, the starting time of an exchange and the instant which ends the exchange of a minimum differ from one element exchanged to another; we agree finally that the exchange of an element continues after the exchange of a minimum form, but ceases as soon as the exchange of its maximum form ends. The exchange of this element can then no longer be repeated without a new command.

The exchange action is totally critical for the parent group.

## B) Derivative (do) action

The instant of command is the instant of derivative activation of the exchange action; the instant of activation is the same as the foregoing; the instant o inhibition is, at the earliest:
(I) the instant of inhibition by abort, of the parent action group of the exchanger group,
(II) the instant of inhibition of its last descendant group.

The derivative action is free for the exchange action and for the parent group of the exchanger action.

## General

This simple example was designed by the EWICS ${ }^{42}$ during the years 80s in order to check the features of descriptions tools and methodologies used in industrial control.

In this example, each primary morphology introduces a list of descendent items which may be,

- either described in the same way, with the same language,
- or described differently.

The «use clause» is available in order to point at these description. This clause is not presented in this example.

## V. 2 Fisrst level of analyse

```
process Weighing_Mixing_System is
    receiver Feed (x : in Power);
    receiver Start (x : in Pulse);
    receiver Stop (x : in Pulse)
    sender issue (x : out Produce);
end Weighing_Mixing_System;
process body Weighing_Mixing_System is
    process Tank_A;
    process Tank_B
    process Store;
    process Weight_Unit;
    process Mixer;
    process Control;
    binder Feeder
    binder Status;
    binder Command
    binder Network;
    binder Fluid;
    binder Solid;
begin
    receive Feed (Energy : in Power)
    do
    repeat
        receive Start (P : in Pulse)
        do
            initiate Feeder
            initiate Tank_A, Tank_B, Store, Weight_Unit, Mixer, Control,
            initiate Status, Command, Network, Fluid, Solid;
        end Start;
        receive Stop (P : in Pulse);
            abort Tank A, Tank B, Store, Mixer, Control,
            abort Status, Command, Network, Fluid, Solid;
            abort Feeder;
            end Stop;
    end repeat;
    end Feed;
end Weighing_Mixing_System
```

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```
-- Analysis, 1st level, visible morphologies
process Tank_A is
    receiver Feed (W : in Power);
    sender Status (S : out Level);
    sender Valve (F : out Fluid_A);
    receiver Command (P : in Raising_Edge);
end Tank_A;
process Tank_B is
    receiver Feed (W : in Power)
    sender Status (S : out Level);
    sender Valve (F : out Fluid_B);
    receiver Command (P : in Raising_Edge);
end Tank_B;
process Store is
    receiver Feed (W : in Power);
    sender Status (S : out Level);
    sender Gate (B : out Bricks);
    receiver Command (P : in Raising_Edge);
    sender Network (D : out Data);
end Store;
process Weight_Unit is
    receiver Feed (W : in Power);
    sender Status (S : out Level);
    receiver Before_Weighing (F : in Fluid);
    sender After_Weighing (F : out Fluid);
    connector Network (D : inout Data);
    receiver Command (P : in Raising_Edge);
end Weight_Unit;
process Mixer is
    receiver Feed (W : in Power);
    sender Status (S : out Level);
    sender Up_Limit (HL : out Level);
    sender Down_Limit (DL : out Level);
    receiver All (X : in Produce);
    sender issue (X : out Produce);
    receiver Up (P : in Raising Edge)
    receiver Down (P : in Raising_Edge);
end Mixer;
process Control is
    receiver Status(1..16) (S : in Level);
    connector Network (D : inout Data);
    sender Command(1..16)(P : out Pulse);
end Control;
binder Feeder is
begin
Tank_A.Feed :=Weighing_Mixing_System. Feed,
Tank_B.Feed :=Weighing_Mixing_System. Feed,
Store.Feed :=Weighing_Mixing_System. Feed,
Weight_Unit.Feed:=Weighing_Mixing_System. Feed,
Mixer.Feed :=Weighing_Mixing_System. Feed,
end Feeder;
binder Status is
Control.Status(1):=Tanker_A.Status,
Control.Status(2):=Tanker_B.Status,
Control.Status(3):=Store.Status,
Control.Status(4):=Weight_Unit.Status,
Control.Status(5):=Mixer.Status,
Control.Status(8):=Mixer.Up_Limit,
Control.Status(9):=Mixer.Down_Limit,
end Status;
binder Command is
Tanker_A.Command
Tanker_B.Command
Store.Command
Weight_Unit.Command
Mixer.Command
Mixer.Up
Mixer.Down
```

:=Control.Command(1), :=Control.Command(2), :=Control.Command(3), :=Control.Command(4), :=Control.Command(5), :=Control.Command(8), :=Control.Command(9),

```
end Command;
binder Network is
Control.Network :=Store.Network,
Control.Network :=Weight_Unit.Network,
end Network;
binder Pipes is
Weigh_t_Unit.Before_Weighing :=Tanker_A.Valve,
Weigh_t_Unit.Before_Weighing :=Tanker_B.Valve,
Mixer.All :=Weight_Unit.After_Weighing,
Weighing_Mixing_System.Issue :=Mixer.Issue,
end Pipes;
binder Belt is
Mixer.All :=Store.Gate,
end Belt;
```


## Second level of analyse

```
process body Control is
```

flux High ;
flux Down ;
flux Bottom_Up ;
flux Top_Down ;
subflux Two is Data
subflux $A$ is Data;
subflux $A B$ is Data;
P1, P2 : Pulse ;

## begin

-- A check and a reset of all the devices must preceed the following phase.
-- The check and reset phase is not described in this simplified example.

## repeat

-- Tank_A.Valve opening
-- Trigger for Tank_A.Valve opening :
send Command(1) (P : out Pulse);
-- Weighing Fluid_A
receive Status (1) (s1 : in Bottom_Up)
do
receive Network (x : in A) -- Waiting for Fluid_A as required
do
send Command(1) (P : out Pulse); -- Trigger for Tank_A.Valve closing
end Network;
end Status;
receive Status(1) (s1 : in Top_Down); -- Waiting for Tank_A closing
-- Tank_B.Valve opening
send Command(2) ( $P$ : out Pulse); -- Trigger for Tank_B.Valve opening
-- Weighing Fluid_B
receive Status (2) (s2 : in Bottom_Up)
do
receive Network ( $x$ : in $A B$ ) -- Waiting for Fluid_B as required
do
send Command( 2) (P : out Pulse); -- Trigger for Tank_B.Valve closing
end Network;
end Status;
receive Status( 2) (s2 : in Top_Down); -- Waiting for Tank_B closing
-- Mixing Fluid_A and Fluid_B
send Command(5) ( P : out Pulse), -- Trigger for Mixer starting
send Command( 3) ( $P$ : out Pulse), -- Trigger

- for Weight_Unit
- after_Weighing opening

```
-- Adding two Bricks
send Command( 3) (P : out Pulse); -- Triger for Store.Gate opening
receive Network (x : in Two)
do
send Command( 3) (P : out Pulse); -- Trigger for Store.Gate closing
end Network;
-- Mixing Fluids and Bricks
begin
    P2 := translate(Timing) P1,
    send Command( 5) (P2 : out Pulse);
    end
-- Mixer emptying
send Command(10) (P : out Pulse)
receive Status(10) (s10 : in Down)
do
    send Command( 9) (P : out Pulse);
end Status;
receive Status( 9) (s09 : in UP);
end repeat;
end Control;
process body Tank_A is
flux Power:
flux Level;
flux Fluid_A
flux Raising_Edge;
subflux On is Level;
subflux Off is Level;
S : Level;
begin
    receive Feed (W : in Power),
    S := Off,
    send Status (S : out Level),
    repeat
        receive Valve (X : in Raising_Edge)
    do
        S := On,
    end Valve
    receive Valve (X : in Raising_Edge)
    do
        S := Off
    end Valve;
    end repeat
end Tank_A;
```


## VI. 1

General
(to be completed later).

## VI. 2

Fisrst level of analyse

```
process ADC is
    receiver Voltage (V : in Signal) ;
    receiver Analog (A : in Signal) ;
    receiver Trigger (Start : in Raising_Edge);
    sender Result (Word : out Parallel_Levels) ;
    sender Status (Busy : out Bivalent, Fault : out Bivalent) ;
end ADC ;
process body ADC is
    flux Signal ;
    subflux Level is Signal ;
    subflux Pulse is Signal ;
    subflux Low is Level;
    subflux High is Level;
    subflux Null is Level;
    subflux Raising_Edge is Pulse;
    flux Bivalent
    is (Low, High)
    flux
    receiver Stop1 (Over : in Pulse) ;
    receiver Stop2 (End : in Pulse) ;
    sender Reset ( }\textrm{R}\mathrm{ : in Pulse) ;
    process Compare ;
    process DAC ;
    process Counter ;
    process Pulse_Generator ;
    process Store ;
```

```
begin
    receive Voltage (V : in Voltage)
    do
    repeat
    receive Trigger (Start : in Pulse)
    do
        initiate Compare, DAC, Counter, Pulse_Generator, Store,
        send Status (Busy : out High, Fault : out Low) ;
        - Busy without failing
        send Reset ( R : out Pulse) ;
        for I in (0..7)
        loop
            Word(I):= Low,
        end loop,
        send Result (Word : out Parallel_Levels),
    end
        Trigger ;
    receive Analog (A : in Signal),
    send Result (Word : out Signal),
    select
        receive Stop1 (Over : in Pulse) ;
        do
            send Status (Busy : out High, Fault : out High) ;
            -- Busy and failing
            abort DAC, Counter ;
        end Stop1
    or
        receive Stop2 (End : in Pulse)
        do
        send Status (Busy : out Low, Fault : out Low) ;
        send Result (Word : out Parallel Levels)
        abort Compare, DAC, Counter, Pulse_Generator, Store ;
        end
    end
    end
end
end
ADC ;
```

```
process Compare is
    connector Plug(1..4) (S : inout Signal) ;
end Compare ;
process body Compare is
    flux Pulse ;
begin
    receive Plug(1) (Ss1 : in Signal) -- receives the voltage supply
    do
    receive Plug(2) (Ss2 : in Signal) -- receives the analog scale
    do
        receive Plug(3) (Ss3 : in Signal) -- receives the analog signal
        do
            repeat
                Sp1 := sample Ss1,
                Sp2 := sample Ss2,
                Sp3 := sample Ss3,
            if
                Sp2>Sp3
            then
                send Plug(4) (Sp1 : out Pulse);
            endif ;
            end repeat ;
        end Plug
    end Plug ;
end Plug ;
end Compare ;
process DAC is
    receiver Voltage (V : in High);
    receiver Digit_DAC (Digit : in Parallel_Levels) ;
    sender Analog_DAC (Scale : out Analog_Scale) ;
end Dac ;
process body DAC is
\begin{tabular}{lll} 
subflux & Level & is Signal ; \\
subflux & Low & is Level; \\
subflux & High & is Level; \\
flux & Parallel_Levels & is array (0..7) of Level \\
& &
\end{tabular}
begin
    receive Voltage (V : in High)
    do
    repeat
        receive Digit_DAC (Digit : in Parallel_Levels)
        do
            send Analog_DAC (Scale : out Analog_Scale) ;
        end Digit_DAC ;
    end repeat ;
    end Voltage ;
end DAC ;
```

```
\begin{tabular}{lllll} 
process & Counter is & & \\
receiver & Voltage & (V & : in Signal) ; \\
receiver & Trigger & (Start & : in Raising_Edge) ; \\
receiver & Reset & (R & : in Raising_Edge) ; \\
receiver & Clock & (String & : in Signal); \\
sender & Binary_Value & (Word & out Parallel_Pulses) ; \\
sender & Full_Scale & (Overflow : out Pulse) ; \\
end & Counter ; & \\
process body Counter is &
\end{tabular}
    flux Pulse ; R Raising_Edge is Pulse ;
    flux Level:
    flux Parallél_Pulses is array (0..8) of Pulse ;
    flux Parallel_Levels is array (0..8) of Level ;
    sender Reset_General (Parallel_R : out Parallel_Pulses) ;
    sender Reset_Element(0..8) (Single_R : out Pulse);
    sender Binary_Element(0..7) (Bit : out Level) ;
    process Flip_Flop (0..8);
    adapter Share_Out ;
    adapter Word_Integrator ;
    binder Voltage_Supply ;
    binder Reset ;
    binder Binary_Count ;
    binder Binary_Store ;
begin
    for I in (0..8)
    loop
    initiate Flip_Flop(I) ,
    end loop ;
    receive Voltage (V : in High)
    do (in High)
    receive Trigger (Start : in Raising_Edge)
    do
        receive Reset (R : in Raising_Edge)
        do
            send Reset_General (Parallel_R : out Pulse) ;
            receive Clock (String : in Signal),
            send Binary_Value (Word : out Parallel_Pulses),
            send Full_Scale (Overflow : out Pulse),
        end Reset ;
    end Trigger ;
    end Voltage ;
end Counter ;
process Pulse_Generator is
    receiver Voltage (V : in High) ;
    sender Clock (S : out Signal) ;
end Pulse_Generator ;
process body Pulse_Generator is
    subflux Pulse_and_Delay is Signal ;
begin
    receive Voltage (V : in High)
do
    repeat
        S := Pulse_and_Delay
        send Clock (S : out Signal) ;
    end repeat
    end Voltage
end Pulse_Genarator ;
```



```
process Flip_Flop is
```



```
begin
\begin{tabular}{lll}
\begin{tabular}{l} 
receive \\
do \\
receive
\end{tabular} & Voltage & (V : in High) ; \\
& Reset & \((R:\) in Raising_Edge)
\end{tabular}
    do
        Value := Zero,
        Bit0 := sample Zero,
        Bit1 := sample One,
        send Binary_Element(0) (Bit0 : out Pulse),
        send Binary_Element(1) (Bit1 : out Pulse),
    end
    Reset ;
        receive Enter
                            (Edge : in Raising_Edge)
        do
            case Value is
            when Value=Zero =>
            Value := One,
            Bit0 := sample Zero,
            Bit1 := sample One,
            send Binary_Element(0) (Bit0 : out Pulse)
            send Binary_Element(1) (Bit1 : out Pulse),
            when others =>
            Value := Zero,
            Bit0 := sample One,
            Bit1 := sample Zero,
            send Binary_Element(0) (Bit0 : out Pulse),
            send Binary_Element(1) (Bit1 : out Pulse)
            end case ;
            end
    end
    end
    Enter ;
    repeat ;
    Voltage ;
end Flip_Flop ;
```

-- Links within ADC
binder Card is
begin
begin

| Compare.Plug(1) | $:=A D C . C o m p a r e(1)$, |
| :--- | :--- |
| Compare.Plug(2) | $:=A D C . A n a l o g$, |
| Compare.Plug(3) | $:=$ DAC.Analog_DAC, |
| DAC.Voltage | $:=A D C . V o l t a g e, ~$ |
| Counter.Voltage | :=ADC.Voltage, |
| Pulse_Generator.Voltage | :=ADC.Voltage, |
| Store.Voltage | :=ADC.Voltage, |
| Counter.Reset | $:=$ ADC.Reset, |
| Counter.Trigger | $:=$ ADC.Trigger, |
| Counter.Clock | $:=$ Pulse_Generator.Clock, |
| DAC.Digit_DAC | $:=$ Counter.Binary_Value, |
| Store.Digit_Store | $:=$ Counter.Binary_Value, |
| ADC.Stop1 | $:=$ Counter.Fullscal, |
| ADC.Stop2 | $:=$ Compare.Plug(4), |
| ADC.Result | $:=$ Store.Result, |

```
end Card ;
```

-- Adapters and links within Counter
adapter Share_Out is
begin
Reset_Element (0).Single_R :=Reset_General.Parallel_R(0),
Reset_Element (1).Single_R :=Reset_General.Parallel_R(1),
Reset_Element (2).Single_R :=Reset_General.Parallel_R(2),
Reset_Element (3).Single_R :=Reset_General.Parallel_R(3),
Reset_Element (4).Single_R :=Reset_General.Parallel_R(4),
Reset_Element (5).Single_R :=Reset_General.Parallel_R(5),
Reset_Element (6).Single_R :=Reset_General.Parallel_R(6),
Reset_Element (7).Single_R :=Reset_General.Parallel_R(7),
Reset_Element (8).Single_R :=Reset_General.Parallel_R(8),
end Share_Out ;
adapter Word_Integrator is
begin
Binary_Value.Word(0) :=Binary_Element(0).Bit,
Binary_Value.Word(1) :=Binary_Element(1).Bit,
Binary_Value.Word(2) :=Binary_Element(2).Bit,
Binary_Value.Word(3) :=Binary_Element(3).Bit,
Binary_Value.Word(4) :=Binary_Element(4).Bit,
Binary_Value.Word(5) :=Binary_Element(5).Bit,
Binary_Value.Word(6) :=Binary_Element(6).Bit,
Binary_Value.Word(7) :=Binary_Element(7).Bit,
end Word_Integrator ;
binder Voltage_Supply is
begin

```
Flip_Flop (0).Voltage:= Counter.Voltage(0),
Flip_Flop (1).Voltage:= Counter.Voltage(1),
Flip_Flop (2).Voltage:= Counter.Voltage(2),
Flip_Flop (3).Voltage:= Counter.Voltage(3),
Flip_Flop (4).Voltage:= Counter.Voltage(4),
Flip_Flop (5).Voltage:= Counter.Voltage(5),
Flip_Flop (6).Voltage:= Counter.Voltage(6),
Flip_Flop (7).Voltage:= Counter.Voltage(7)
Flip_Flop (8).Voltage:= Counter.Voltage(8),
end Voltage_Supply ;
```

```
binder Reset is
begin
    Flip_Flop (0).Reset:= Counter.Reset_Element(0),
    Flip_Flop (1).Reset:= Counter.Reset_Element(1)
    Flip_Flop (2).Reset:= Counter.Reset_Element(2),
    Flip_Flop (3).Reset:= Counter.Reset_Element(3),
    Flip_Flop (4).Reset:= Counter.Reset_Element(4),
    Flip_Flop (5).Reset:= Counter.Reset_Element(5)
    Flip_Flop (6).Reset:= Counter.Reset_Element(6)
    Flip_Flop (7).Reset:= Counter.Reset_Element(7),
    Flip_Flop (8).Reset:= Counter.Reset_Element(8),
```

end Reset ;
binder Binary_Count is
begin
Flip_Flop (0).Enter:= Counter.Clock,
Flip_Flop (1).Enter:= Flip_Flop(0).Binary_Element(0),
Flip_Flop (2).Enter:= Flip_Flop(1).Binary_Element(0),
Flip_Flop (3).Enter:= Flip_Flop(2).Binary_Element(0),
Flip_Flop (4).Enter:= Flip_Flop(3).Binary_Element(0),
Flip_Flop (5).Enter:= Flip_Flop(4).Binary_Element(0),
Flip_Flop (6).Enter:= Flip_Flop(5).Binary_Element(0),
Flip_Flop (7).Enter:= Flip_Flop(6).Binary_Element(0),
Flip_Flop (8).Enter:= Flip_Flop(7).Binary_Element(0),
end Local ;
binder Binary_Store is
begin

Counter.Binary_Element (0):= Flip_Flop(0).Binary_Element(1), Counter.Binary_Element (1):= Flip_Flop(1).Binary_Element(1), Counter.Binary_Element (2):= Flip_Flop(2).Binary_Element(1), Counter.Binary_Element (3):= Flip_Flop(3).Binary_Element(1), Counter.Binary_Element (4):= Flip_Flop(4).Binary_Element(1), Counter.Binary_Element (5):= Flip_Flop(5).Binary_Element(1), Counter.Binary_Element (6):= Flip_Flop(6).Binary_Element(1), Counter.Binary_Element (7):= Flip_Flop(7).Binary_Element(1), Counter.Full_Scale $:=$ Flip_Flop(8).Binary_Element(1), -- Counter overflow
end Binary_Store ;

## General

Ayeb (B. el). Toward Systematic Construction of Diagnogstic Systems for Large Industrial Plants: Methods, Languages, and Tools. in: IEEE Trans. on Knowledge and data Engineering. Oct. 1994, vol.6, n5, ISSN 1041-4347. pp.698/712.
Battiston (E.), Cindio (F. de), Mauri (G.). OBJSA nets: a class of high-level nets having objects as domains. in: Advances in Petri nets. Springer-Verlag, Berlin, 1988. pp.20/43. ISBN: 3 540505806.

| xix | Category Theory Applied to Computation and Control. Edited by G. Goos and J. Hartmanis. Lecture Notes in Computer Science. Springer-Verlag, Berlin, ..., 1975. 246p. ISBN 3-540-07142-3. |
| :---: | :---: |
| XX | Haurat (A.), Piard (F.). The « OLYMPIOS » model: an algebraic specification for modelling the information system of a manufacturing enterprise. in: 1993 CompEuro Proceedings. Computer in Design, Manufacturing, and Production. IEEE, Comp. Soc. Press., Los Alamitos, 1993. pp.330/335. ISBN: 0818640308. |
| xxi | Hotaka (Ryosuke), Björn (Michael). Data oriented Approach to Business Information Modelling. Univ. of Tsukuba, 1993. 19p. |
| xxii | Jensen (K.). An introduction to the theoretical aspects of coloured Petri nets. in: Decade of Concurrency. Reflections and Perspectives. REX School/Symposium Proceedings. 1-4 June 1993. Springer-Verlag, Berlin, 1994. pp230/272. ISBN 3540580433. |
| xxiii | Korff (M.). Single pushout transformation of equationally defined graph structures with applications to actor systems. In: Graph Transformations in Computer Science. International Workshop Proceedings. 4-8 Jan. 1993. Springer-Verlag, Berlin, 1994. pp.234/247. ISBN: 3 540577874. |
| xxiv | Lahdelma (R.). An object-oriented mathematical modelling system.. in: Acta Polytechnica Scandinavica, Mathematics and Computer Science Series, n ${ }^{\circ}$ MA66. pp.1/77. ISSN: 03552713. |
| xxv | Lieberherr (Karl, J.), Xiao (Cun). Object-Oriented Software Evolution. in: IEEE transactions on software engineering. April 1993, vol.19, nº4. (ISSN 0098-5589). pp.313/343. |
| xxvi | McLane (S.), Birkhoff (G.). Algèbre. Tome I. Structures fondamentales. Traduit de l'américain par J. Weil. Préface de J. Dieudonné. Nouveau Tirage. Cahiers Scientifiques. Publiés Sous la Direction De M. Gaston Julia. Fascicule XXXV. Gauthier-Villars, Paris, 1971. xxivp., 410p. |
| xxvii | McLane (S.), Birkhoff (G.). Algèbre. Tome II. Les grands théorèmes. Traduit de l'américain par J. Weil. Préface de J. Dieudonné. Nouveau Tirage. Cahiers Scientifiques. Publiés Sous la Direction De M. Gaston Julia. Fascicule XXXVI. Gauthier-Villars, Paris, 1971. xivp., 344p. |
| xxviii | Najm (E.), Budkowski (S.), Gilot (T.), Lumbroso (L.). General presentation of SCAN. A distributed system modelling and validation tool. Agence de l'Informatique. Paris la Défense, France. North-Holland, Amsterdam, 1986. xip., 544p. ISBN: 0444878815. |
| xxix | Noaks (D. R.), Wood (K.). System modelling for safety and fault analysis using the software tool NP-Circuit. in: IEE Colloquium on « Structured Methods for Hardware Systems ». Digest N¹994/110. IEE, London, 1994. |
| xxx | Phoa (W.), Fourman (M.). A proposal categorical semantics for pure ML. in: Automata, Languages and Programming. $19^{\text {th }}$ International Colloquium Proceedings. Springer-Verlag, Berlin, 1992. pp.533/544. |
| xxxi | Rattray (C.). The shape of complex systems. Computer Aided System Theory. EUROCAST'93. Third International Workshop on Computer Aided System Theory Proceedings; 22-26 Feb. 1993. Springer-Verlag, Berlin, 1994. pp.72/82. ISBN: 3540576010. |
| xxxii | Reggio (G.). Event logic for specifying dynamic data types. in: Recent Trends in Data Type Specification. $8^{\text {th }}$ Workshop on Specification on Abstract Data Types joint with $3^{\text {rd }}$ COMPASS Workshop. Selected papers. Springer Verlag, Berlin, 1993. pp. 292/309. ISBN: 3540556379 2. |
| xxxiii | Savoysky (S.). Description sommaire d'une méthode d'utilisation de l'algèbre multilinéaire pour la représentation de systèmes. in: Réflexions sur de nouvelles approches dans l'étude des systèmes. Actes du Coll. org. à Paris, les 10, 11, et 12 juin 1975, par l'Ec. centr. des arts et man. et l'Ec. nat. sup. de techn. av.,... Association nationale de la recherche technique, Paris, 1975. pp. 177/200. |
| xxxiv | Savoysky (S.). Analysis and description of automatic control system. in: Real Time Programming 1980. Proc. of the IFAC/IFIP Workshop. Schloss Retzhof, Leibnitz, Austria, 14-16 April 1980. Ed. by V.H. Haase. Pergamon Press, Oxford,..., 1980. ISBN 008027305 X. pp. 45/55 |
| xxxv | Savoysky (S.). The Use of Ada for the Specification of Automata in Civil Engineering. in: Real Time Programming 1981. Proc. of the IFAC/IFIP Work. Kyoto, Japan, 31 August - 2 Sept. 1981. Pergamon Press, Oxford,..., 1982. ISBN 0-08-027613-X. pp. 129/138. |


| xxxvi | Savoysky (S.). Specification of exchange mechanisms between elements of industrial systems. in: IECON'85, Proceedings, Industrial applications of mini, micro, and personnal computers. San Francisco, Ca., nov. 18-22, 1985., vol.2. IEEE, sl., 1985. IEEE Cat. Numb. 85CH2160-0. Libr. of Congr. Cat. Numb. 85-60211. pp. 794/799. |
| :---: | :---: |
| xxxvii | Sowa (John F.). Knowledge Representation. Logical, Philosophical, ad Computational Foundations. July 17th, 1995. Draft of a book in preparation.423p. |
| xxxviii | Thom (René). La théorie des catastrophes. in: Modèles mathématiques de la morphogenèse. Chr. Bourgoi Ed., Paris, 1908. pp.81/90. |
| xxxix | Tucker (J.V.), ZUCKER (J.I.). Toward a general theory of computation and specification over abstract data types. in: Advances in Computing and Information. ICCI'90. International Conference Proceedings. Springer-Verlag, Berlin, 1990. pp.129/133. ISBN 3540535047. |
| xl | Tuzhilin (A.A.). Category theory of structural sets with application to mathematical modelling and systems analysis. in: Mathematical Modelling, vol.7, $n^{\circ} 1$. pp.27/48. ISSN: 0270-0255. |
| xli | Vries (J.A. de), Breedveld (P.C.), Meindertsma (P.). Polymorphic modelling of engineering systems. in: International Conference on Bond Graph Modelling ICBGM'93. 1993 Western Simulation Multiconference. SCS, San Diego (CA), 1993. pp.17/22. |
| xlii | Zhao Xudong, Feng Yulin. Automatic and hierarchical verification for concurrent systems. in: Journal of Computer Science and Technology (E), vol.5, n³3, pp.241/249. China, 1990. ISSN: 10009000. |
| VII. 2 | CSMF, working drafts |
| CSMF ABQ-010 | US Contribution on Abstract Conceptual Schema Language (ACSL) Syntax and Semantics [SC21/WG3 N1781 Clause 8.2, revised Clause 8.1]. |
| CSMF ABQ-015 | OEII (J.L.H.). A Meta Model Transformation approach towards harmonisation in information systems modelling. Nov. 2, 1994. |
| CSMF ABQ-024 | Wielinga (Bob) and al. Framework and Formalism for Expressing Ontologies. KACTUS. Esprit Project 8145. Univ. of Amsterdam, 1994. |
| CSMF AIX-007 | $\Rightarrow$ ISO/IEC TC97/SC5/WG3 |
| CSMF AIX-008 | $\Rightarrow$ ISO/TC 184/SC4 DIS 10303-11 |
| CSMF AIX-009 | $\Rightarrow$ ISO/IEC JTC1/SC21 |
| CSMF SOU-007 | A Data Modelling Facility: JDMF/MODEL-1992 R1.1. |

## VII. 3

AFNOR
CGTI/CN21
F2236
ANSI/MIL-STD
1815 A
BSI
DD 210: 1992
EWOS/ETG
012
IRDS
part 2 93-196
IRDS
part 193-196
ISO/IEC
TC97/SC5/WG3 9007

## Normative documents

AFNOR CGTI/CN21 F2236. A direct computational language semantics for Part 4 of the RM-ODP. AFNOR, Paris, 1994/07/05. 12p.
ANSI. Reference manual for the Ada ${ }^{\circledR}$ programming language. Alsys, La Celle-Saint-Cloud, 1983.

BSI. Guide to A framework for user requirements for Information Technology. Draft for Development. DD 210: 1992. 23p.
EWOS. Guide to profiles for the open system environement. Approved by EWOS/TA (1991)..
IRDS. Conceptual Schema. Part 2: Modelling Language Analysis. X3H4/93-196. xp., 98p.
IRDS. Conceptual Schema. Part 1: Conceptual Schema for IRDS. X3H4/936196. xp., 160p.
ISO. Concepts and terminology for the conceptual schema and the information base. ISO TR 9007.

ISO
TC184/SC4
WG3
N103

ISO
TC184/SC5/
WG4
N116
ISO/IEC
IS
10746-2
ISO/IEC
IS
10746-3
ISO/IEC JTC
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ISO/IEC JTC1
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ISO/IEC JTC1 N
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ISO/IEC
JTC1/SC21/
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JTC1/SC21/
WG3
1866
ISO/IEC
JTC1/SC7/
WG11
BRI22

ISO/TC 184/SC4 DIS 10303-11

SC2 1 N7054

ISO TC184/SC4/WG3. Technical Report on the Semantic Unification Meta-Model. Volume 1: Semantic Unification of Static Models. A technical report of the Dictionary : Methodology Committee of the IGES/PDE Organization on the application of formal semantics and symbolic logic to the integration and unification of models. Prepared by the Dictionary / Methodology Committee of the IGES / PDES Organization. Version 1.0. October 19, 1992. N103. 124p., Appendix,
ISO TC184/SC5/WG4. Industrial Atomation Systems - MAPLE Architecture. Draft of Committee Draft. for DIS Balloting. May 1995.

ISO/IEC International Standard 10746-2. Information Technology. Open Distributed Processing. Reference Model. - Part 2: Foundations. [IUT Recommendation X.902].

ISO/IEC International Standard 10746-3. Information Technology. Open Distributed Processing. Reference Model. - Part 3: Architecture. [IUT Recommendation X.902].Draft.

ISO/IEC JTC1 TC 22. N1712. Draft Technical Report for EXTensions for Real-time Ada. Work Item Number: JTC 1.22.35. ISO/IEC TR 11735. October 30, 1994. 69p.
ISO/IEC JTC1 TSG 1. Standards necessary to define Interfaces for Application Portability. (IAP). Final report. April 1993. 69p.
ISO/IEC JTC1. Strategic plan for information technology standardisation. A Presentation of the Strategic Objective, Assumptions, Trnds and Needs. SPRG N31. 1 November1994. 4p. ISO/IEC JTC1/SC21. Information Retrieval, Transfer and Management for OSI. N236.

ISO/IEC JTC1/SC21. Information Technology. Open Distributed Processing. Reference Model .- Part 1: Overview and Guide to Use. N8218.

ISO/IEC JTC1/SC21. LOTOS. A formal Description Technique Based on the Temporal Ordering of Observational Behaviour. Amd 1: G-LOTOS. N8913. 118p.
ISO/IEC JTC1/SC21. Information Technology. Open Distributed Processing. Reference Model. - Part 4: Architectural Semantics. N9563.

ISO/IEC JTC1/SC21/WG1. Enhancements to LOTOS. Working Draft on Enhancements to LOTOS. Project 1.21.20.2.3. October 1994.. N1349. Pag. mult.

ISO/IEC JTC1/SC21/WG3. Conceptual Graphs, A presnetation Language for Knowledge in Conceptual Models.. WD of Proposed American National Standard. X3 Project N ${ }^{\circ}$. 1059-D. May 10, 1995. N1865.
ISO/IEC JTC1/SC21/WG3. Knowledge Interchange Format Reference Manual.. WD for an ANS. N1866.

ISO/IEC JTC1/SC7/WG11. Fulton (Dr. James A.) Strategy for the Integration of Know-ledge-Based Engineering Data. Boeing Information and Support Services Research and Technology. Working Draft 1.2, June 19, 1995. 22p. ISO TC184/SC4/WG5 N234, ISO/IEC JTC1/SC7/WG11 BRI22. (OTT20).
ISO/TC184/SC4. Industrial automation systems - Product data representation and exchange Part 11: Description method: The EXPRESS language reference manual. XIVp., 110p. ISO/DIS 10303-11.
SC21. Basic Reference Model of Open Distributed Processing - Part 2: Descriptive Model.

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## BIography

Born in 1933, Serge Savoysky earned his «Licence ès Science» in 1956 and he received his State Engineer degree in 1957 from the «Ecole Nationale Supérieure de Mécanique et d'Aérotechnique»; he received later his French State «Doctorat ès Sciences, Mathématiques» from the «Université Pierre et Marie Curie, Paris» in 1983. He first entered as engineer in the Flying Trying Centre of the French Air Forces in 1957. During his military duty, he continued to serve as officer in these Air Forces (1957/1960), in Algeria. In 1960, he entered a Building Design Office again as engineer and he introduced in this company the data processing technologies. In 1970 he became the first head of Data Processing Department in the «Laboratoire Central
 des Ponts et Chausées », main research centre of the French State Roads and Bridges Laboratories Network. Since this time until 1992, he managed data processing in this network, mainly for laboratory and worksite process control, and conducted research in this domain where he met the subject of his doctorate thesis : Proposal of Theoretical Element for System Description; his personal main interest was the formal description of industrial systems. During the same time, he taught process control technologies in «Ecole Nationale des Travaux Publics de l'Etat». Now retired, he continues to investigate in the field of formal system description methodologies. He enjoys collecting calculating machines and apparatus and books of the past. He is married and has two adult children.


[^0]:    1 The L.C.P.C. is the Main Road and Bridge Research Center of the French Ministry of Equipment.
    2 Highway Engineering Office.
    3 European Bull Users Association.
    4 Savoysky: 1975, xxxiii ; 1981, xxxv.

[^1]:    10 The notions of morphology and physiology are commonplace as concerns living systems. Maurice d'Ocagne was apparently the first to explicitly transpose these notions to the field of mechanical calculation while commenting the thesis of Louis Couffignal (in: Histoire abrégée des sciences mathématiques, Vuibert, Paris, 1952, pp. 389/391). These notions are clearly present in information technology, especially in the software industry; usually, they are however distinguished other than by their usual names.
    11 The existence of such information system is possibly the best reason for arguing in favor of the integration of this study within the TC21 WG3 activity.

[^2]:    19 Algebraically, the diagram of Figure 5 "commutes": $\mathrm{O}=\mathrm{V}-1\left(\mathrm{M}^{\prime}(\mathrm{T}(\mathrm{I}))=\mathrm{M}(\mathrm{I})\right.$ and $\mathrm{M}^{\prime}=\mathrm{U}(\mathrm{M})$. This strictly theoretical changeover is in practice not feasible; the approximate result $\mathrm{O}^{*}=\mathrm{V}-1\left(\mathrm{O}^{\prime}\right)$ must then be compared with the template of O. Algebra makes it possible to rigorously express these conditions and then to construct the formal verification algorithms. This observation, which would appear of no value in the case of simple figures, takes its full importance in the case of functionalities obtained by combinations.
    20 This basic diagram may also represent a relative definition of the notions of user and supplier: the user is the individual or organisation which specifies a functionality and, consequently, requires a service; the supplier is the one who proposes and constructs the solution providing the required service. Such a diagram can be used iteratively: according to this remark, there is no absolute definition of user or supplier; in particular, the notion of end user is always a conventional one.

[^3]:    23 Any operating recurrence of a system, a machine or a produce is a phenomenon of the physical world; the process is its particular model.
    24 The theory of distributions was devised to obviate this kind of difficulty.

[^4]:    31 This proposition is a simplified example and not an exhaustive study of the subject.

[^5]:    35 M . Bézout. Théorie générale des équations algèbriques. Préface. 1779
    36 Laurent Schwartz. Cours d'analyse. Ecole Polytechnique.

[^6]:    37 Within the framework of the general theory of categories, this is a "small category".
    38 There are restrictions in the definition of a category: category theory uses the notion of "class" which is more general than the notion of "set" and extends the notion of function to classes.

